Probabilistic analysis in slope stability

Analyses Probabilistes de stabilité des pentes

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ABSTRACT

Geotechnical design is, perhaps, the civil engineering subject most dominated by uncertainty. Although partly due to limited field investigations, the main reason of this uncertainty is the highly heterogeneous and anisotropic nature of ground materials. Deterministic design methods as required by design codes, attempt to account for ground uncertainty by adopting conservative values of ground parameters and relatively large safety factors. This paper performs probabilistic analyses of soil slopes using Monte Carlo simulation in order to investigate the effect of these assumptions on the calculated probability of failure of the slopes. For slopes in saturated cohesive materials under undrained conditions an analytical solution is proposed while for drained conditions parametric analyses are performed. It is shown that slopes designed according to Eurocode 7 (EN1997-1) correspond to probabilities of failure up to 12%. Based on the results of these analyses, diagrams are proposed for the estimation of the probability of failure as a function of the geometrical, geotechnical and support parameters of the slope.

RÉSUMÉ

Le calcul géotechnique est probablement l'endroit le plus incertain de génie civil à cause de l'hétérogénéité et l'anisotropie du sol. Les calculs déterministes, après les demandes des codes et règlements, adoptent des valeurs conservatrices pour les paramètres du sol et des facteurs de sécurité assez élevés, pour dépasser ces incertitudes. L'article performe des analyses probabilistes de stabilité des pentes avec la simulation de Monte Carlo pour étudier l'approche précédente. Dans un tel contexte, la stabilité des pentes dans un milieu saturé et cohésive se divise en conditions non drainé, où une solution analytique est proposée, et en conditions drainé, avec des analyses paramétriques. Il est montré que la stabilité des pentes en fonction de Eurocode 7 (EN1997-1) donne de probabilité d'échec de 12%. Basé sur les résultats, des diagrammes sont proposés pour l'estimation de la probabilité d'échec en fonction des paramètres géométrique, géotechnique et soutènement de la pente.

Keywords : Probabilistic analysis, Slope stability, Eurocode 7, Monte Carlo Simulation

1 INTRODUCTION

Despite the uncertainties involved in slope stability problems, the civil engineering profession has been slow in adopting probabilistic techniques (e.g. El-Ramly et al. 2002). All design codes, including Eurocode 7 (2004), attempt to account for ground uncertainty by adopting conservative (characteristic) values of ground parameters and relatively large safety factors.

In order to investigate the stochastic characteristics of slopes designed according to the methodology proposed by Eurocode 7, parametric analyses are performed for a very wide range of geometrical, geotechnical and support parameters of soil slopes. The investigated slopes have height and inclination which provides marginally adequate safety according to the requirements of Eurocode 7 for the ultimate limit state (ULS) of overall stability, using planar failure surfaces and the Mohr-Coulomb failure criterion.

Since slope failure on a planar surface is a force equilibrium problem the distributed forces acting on the slope surface (typically support measures such as nails, anchors, geotextiles, etc.) can be represented by a single concentrated force (F).

Ground strength parameters are considered as random variables following truncated normal distribution. Cohesion and friction angle are considered to be statistically uncorrelated. Unit weight, geometrical and support parameters are considered as deterministic variables.

A probabilistic analysis is then performed for these slopes using Monte Carlo simulation to determine the probability distribution of the safety factor and the corresponding probability of failure (p_f) of the slope (i.e., when the safety factor is less than unity) as a function of the problem parameters.

For the case of slopes in saturated cohesive soil under undrained conditions, an analytical solution is presented for the calculation of the probability of failure.

2 SLOPE DESIGN ACCORDING TO EUROCODE 7

According to Eurocode 7, the overall stability of slopes must be verified in ultimate limit states GEO and STR using one of the three Design Approaches (DA). In Greece, Design Approach 3 (DA-3) is used for slope stability problems (National Annex 2007). The relevant partial factors are presented in Table 1.

Table 1. Partial factors of safety according to Eurocode 7 (DA-3)

	Set	Partial factor
Permanent favourable actions, γ_G	A2	1.00
Angle of shearing resistance, $\gamma_{\varphi'}$ (applied to $\tan\varphi'$)	M2	1.25
Effective cohesion, $\gamma_{c'}$	M2	1.25
Undrained shear strength, γ_{cu}	M2	1.40
Earth resistance, $\gamma_{R;e}$	R3	1.00
Model uncertainty factor*, γ_M		1.10

* for usual unfavourable hydraulic conditions (National Annex 2007)

Since slope stability problems usually involve a large volume of soil, failure depends on the distribution of the mean values of soil strength parameters (Frank et al 2004). Therefore

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the relevant characteristic values are determined from the following formulae (Schneider 1999).

$$c'_k = m_{c'} - 0.50\, V_{c'}\, m_{c'} \tag{1}$$

$$\varphi'_k = m_{\varphi'} - 0.50\, V_{\varphi'}\, m_{\varphi'} \tag{2}$$

$$c_{u,k} = m_{cu} - 0.50\, V_{cu}\, m_{cu} \tag{3}$$

where $m_{c'}$, $m_{\varphi'}$, m_{cu} are the mean values and $V_{c'}$, $V_{\varphi'}$, V_{cu} the coefficients of variation of the geotechnical strength parameters.

The probabilistic analysis proposed is based on the conservative assumption that the standard deviation of the mean value of soil strength parameters equals the standard deviation of the original variable. This assumption results in larger probability of failure and has been adopted because it is difficult and computationally very cumbersome to analytically consider spatial averaging in parametric analyses.

According to Eurocode 7, the design parameters entering the calculations are obtained by dividing the characteristic values by the corresponding partial factors $\gamma_{\varphi'}$, $\gamma_{c'}$ and γ_{cu} .

$$c'_d = c'_k/\gamma_{c'} \tag{4}$$

$$\varphi'_d = \varphi'_k/\gamma_{\varphi'} \tag{5}$$

$$c_{u,d} = c_{u,k}/\gamma_{cu} \tag{6}$$

3 SLOPES IN SOIL

A wide range of slopes was analysed using the following ranges of geometrical, geotechnical and support parameters :

Table 2. Range of analysed parameters

Parameters	Range of values	Number of values
Slope height, H, m	5-40	8
Soil unit weight, γ, kN/m^3	21	1
Cohesion mean value, $m_{c'}$, kPa	0-150	31
Friction angle mean value, $m_{\varphi'}$, deg	20-40	11
Force parameter $F/(\gamma H^2)$	0-0.15	7
Number of slopes examined		19096

Assuming a planar failure surface at an inclination (θ) with respect to the horizontal, the factor of safety (FS) is calculated from Equation 7 (Duncan & Wright 2005).

$$FS = \frac{2\cdot c'\dfrac{H}{\sin\theta} + \gamma H^2\,\dfrac{\sin(\beta-\theta)}{\sin\beta}\cdot\dfrac{\tan\varphi'}{\tan\theta} + 2F\sin(\theta+\omega)\tan\varphi'}{\gamma\cdot H^2\cdot\dfrac{\sin(\beta-\theta)}{\sin\beta} - 2F\cos(\theta+\omega)} \tag{7}$$

where (β) is the slope inclination and (ω) is the angle of the support force (F).

The analysis procedure is illustrated in Figure 1. FS_{det} is the deterministic value of the safety factor calculated from Equation 7 using the design values of all parameters.

The adopted coefficients of variation of the ground parameters are: cohesion $V_{c'}$ = 0.40 (Schultze 1972), friction angle $V_{\varphi'}$=0.12 (Fredlund & Dahlman 1972).

Among the analysed slopes, 10890 are marginally safe according to Eurocode 7, i.e., they satisfy the relationship $FS_{det}=\gamma_M$. For these slopes, a probabilistic analysis is performed using Monte Carlo simulation to determine the probability of failure.

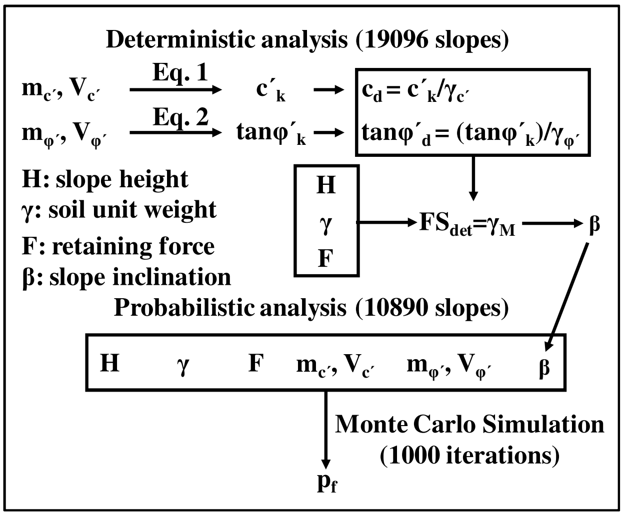

Figure 1. Analysis flow chart

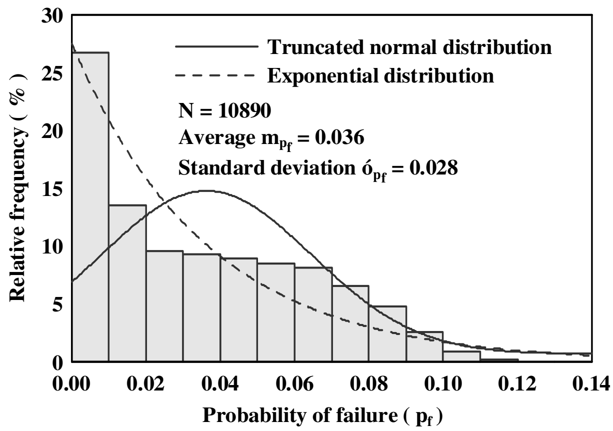

Figure 2. Distribution of the probability of failure for slopes in soil

Figure 2 presents the distribution of p_f for all slopes analysed. The range of p_f is very large (0-12%) considering that all slopes are marginally safe according to the same deterministic methodology (Eurocode 7). The probability of failure is not uniformly distributed in this range but appears to follow an exponential distribution. Furthermore, about 30% of the slopes have $p_f > 5\%$, a value which can be considered as an acceptable limit.

It was thus attempted to correlate the calculated p_f with the following normalized parameters for the cohesion : $m_{c'}/\gamma H$ and the support force : $F/\gamma H^2$, while the friction angle is considered to be statistically insignificant. Table 3 shows the calculated correlation coefficients:

Table 3. Correlation coefficients of p_f and input parameters

	tanmφ'	$mc'/\gamma H$	$F/\gamma H^2$
p_f	-0.25	0.91	-0.48

The best fit was obtained for the following formula as shown in Figure 3.

$$p_f = 0.087 + 0.33\,(1-F/\gamma H^2)\,(m_{c'}/\gamma H) \tag{8}$$

The above relationship shows that an increase of the support force decreases the probability of failure of the slope, because the support force is a deterministic favourable action. On the contrary, an increase of the cohesion increases p_f because cohesion has high variability. Therefore, slopes in overconsolidated plastic clays have higher p_f than slopes in sandy or gravely clays when designed with the same deterministic methodology.

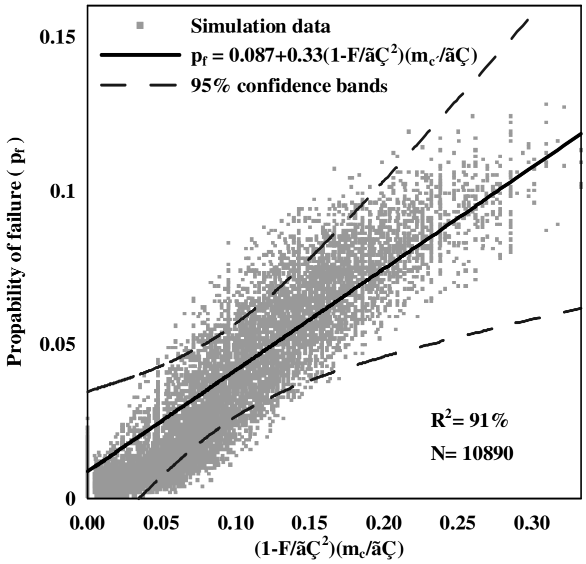

Figure 3. Correlation of the calculated probability of failure (p_f) of 10890 typical slopes in soil with input strength (m_c) and support (F) parameters

4 SOILS UNDER UNDRAINED CONDITIONS

Short-term stability of slopes in fully saturated clays is controlled by the undrained shear strength (c_u) and is commonly analysed using total stresses: $c=c_u$ and $\varphi_u=0$. The undrained shear strength of clays has relatively high variability of the order $V_{cu}=0.40$ (Harr 1987; Kulhawy 1992).

The deterministic factor of safety (FS_{det}) under undrained conditions assuming a planar failure surface is given by the formula (Duncan & Wright 2005):

$$FS_{det} = \frac{2 \cdot \dfrac{c_{u,k}}{\gamma_{cu}} \cdot \dfrac{1}{\gamma H}}{\dfrac{\sin(\beta-\theta)\cdot\sin\theta}{\sin\beta} - \dfrac{F}{\gamma H^2}\cdot\sin 2\theta} \tag{9}$$

Using Monte Carlo simulation, as above, the correlation factors between the probability of failure and the dimensionless parameters $m_{cu}/\gamma H$ and $F/\gamma H^2$ are -0,03 and 0,02 respectively, indicating that the mean value of the undrained shear strength (m_{cu}) and the stabilizing force (F) are not correlated with the probability of failure of the slope.

The critical angle of the failure plane (θ_{cr}) is obtained by setting the derivative of FS with respect to the angle θ equal to zero, which gives:

$$\frac{\sin(\beta - 2\theta_{cr})}{\sin\beta\cdot\cos 2\theta_{cr}} = \frac{2F}{\gamma H^2} \tag{10}$$

Consequently, the critical angle θ_{cr} does not depend on the undrained shear strength c_u and it is a deterministic variable.

By setting the denominator of Equation 9 equal to A, the total deterministic factor of safety ($FS_{det,total}$) is :

$$FS_{det,total} = \frac{2c_{u,k}}{A\gamma H} \tag{11}$$

i.e., the factor of safety is proportional to the random variable c_u while all other dependencies are deterministic. Hence, it is feasible to express the stochastic factor of safety through an analytical solution.

The total stochastic factor of safety ($FS_{st,total}$) is expressed by using undrained shear strength c_u as a stochastic variable which follows the truncated normal distribution :

$$FS_{st,total} = \frac{2c_u}{A\gamma H} \tag{12}$$

By replacing the value A from Equation 11, Equation 12 gives :

$$FS_{st,total} = \frac{FS_{det,total}}{c_{u,k}} \quad c_u \tag{13}$$

The standard deviation of the undrained shear strength is expressed as:

$$\sigma_{cu} = V_{cu}\cdot m_{cu} \tag{14}$$

Finally, the expression of the $FS_{st,total}$ using Equations 3 and 14 is :

$$FS_{st,total} = \frac{FS_{det,total}}{m_{cu}(1-0.5\cdot V_{cu})}\cdot c_u \tag{15}$$

According to the principles of statistics, $FS_{st,total}$ follows normal distribution as it is a linear combination of the stochastic variable c_u . The mean value ($mFS_{st,total}$) and the standard deviation ($\sigma FS_{st,total}$) of $FS_{st,total}$ are:

$$mFS_{st,total} = \frac{FS_{det,total}}{1-0.5V_{cu}} \tag{16}$$

$$\sigma FS_{st,total} = \frac{FS_{det,total}\cdot V_{cu}}{1-0.5V_{cu}} \tag{17}$$

The probability of failure of slopes in cohesive materials under undrained conditions is expressed as:

$$p_f = p(FS_{st,total} < FS_{min} = 1) \tag{18}$$

Transforming in standard normal distribution and using Equations 16 and 17, the expression of p_f is:

$$p_f = \mathrm{Erf}\left(\frac{1-0.5V_{cu} - FS_{det,total}}{FS_{det,total}\cdot V_{cu}}\right) \tag{19}$$

The relationship between the p_f and the $FS_{det,total}$ for various values of V_{cu} is illustrated in Figure 4. It is concluded that in this case, p_f is determined only by the value of $FS_{det,total}$ and V_{cu} which depends on the heterogeneity of the geomaterial.

It is observed that an increase of the value of $FS_{det,total}$ results in a significant decrease of the value of p_f for $FS_{det,total}<2$ and does not induce a considerable decrease in p_f for $FS_{det,total}>2$, because of the decreasing inclination of the curves.

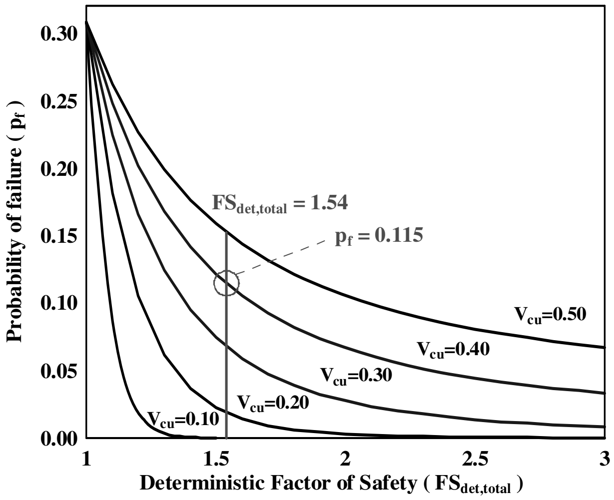

Figure 4. Probability of failure as a function of the deterministic factor of safety (FS_{det}) and the variability index of c_u (V_{cu}) for slopes in cohesive soil under undrained conditions

According to Eurocode 7, $FS_{det,total}$ is equal to $\gamma_M*\gamma_{cu}$=1.1*1.4=1.54. Thus, all slopes that are designed to the ultimate limit state according to Eurocode 7 and have V_{cu}=0.40 are characterized by a probability of failure equal to 11.5%, independently of other characteristics.

5 CONCLUSIONS

The level of safety in geotechnical projects, usually expressed by the Safety Factor in Design Codes, can be better described by the probability of failure using stochastic analyses which account directly for problem uncertainties. Geotechnical problems can be analysed stochastically without added difficulties, since such analyses do not require more data, time and effort (El-Ramly et al. 2002).

The present paper investigates the stochastic characteristics of soil slopes designed according to Eurocode 7 using parametric probabilistic analyses. A closed-form analytical solution is also obtained for undrained analyses of slopes in cohesive materials.

It is shown that the probability of failure of soil slopes, in general, can be estimated from the geometrical parameters of the problem, the unit weight and cohesion of the material and the value of the retaining force. The probability of failure, for the same value of the deterministic Safety Factor ($FS_{det,total}$) can be decreased with the use of support elements (e.g. anchors, geotextiles etc) which are characterized by a very low strength variability. For the slopes designed according to Eurocode 7 probability of failure varies from 0 to 12% percent, while 30% of the slopes correspond to values of p_f larger than 5%.

In the case of undrained analyses, the Safety Factor can be calculated by an analytical expression and is proportional to the undrained shear strength (c_u). It is thus possible to determine the required deterministic Safety Factor ($FS_{det,total}$) for a given probability of failure (p_f) and vice versa. It is shown that for slopes designed according to Eurocode 7, the probability of failure under undrained conditions is 11.5% (V_{cu}=0.40).

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