The experimental determination of the angle of dilatancy in soils

La détermination expérimentale de l'angle de dilatation des sols

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ABSTRACT

The effective shear strength of overconsolidated soils (OCR ≥ 3) without cementation among particles or aggregates of particles is frequently described in terms of the cohesion and the angle of internal friction. An alternative, that is more consistent with the physical behaviour of these materials when sheared (a frictional behaviour) and which is strictly related to the volumetric strains, makes use of the angle of dilatancy and the effective angle of friction.

In practical applications concerning the types of soils mentioned above, the elastic perfectly plastic non-associated Mohr-Coulomb model is still widely used and a quantification of the angle of dilatancy through soil testing is then necessary. Nevertheless this quantification is not always obvious due to ambiguous definitions present in the literature.

The paper deals with the interpretation of the results for different types of soil tests, focusing on the procedures to correctly evaluate the angle of dilatancy. Some cases are presented and discussed.

RÉSUMÉ

La résistance au cisaillement effective des sols sur consolidés (degré de consolidation ≥ 3) sans cimentation entre particules ou des agrégats de particules est fréquemment décrit en termes de cohésion et d'angle de frottement interne. Une alternative plus consistent avec le comportement physique de ces matériaux quand ils sont cisaillés (un comportement complètement contrôlé par le frottement) et qui dépend strictement des déformations volumétriques c'est de faire use de l'angle de dilatation et l'angle de frottement interne.

Dans les applications pratiques avec des sols mentionnés, le modèle élastique parfaitement plastique non associée de Mohr-Coulomb est très appliquée et pour cela, il est nécessaire déterminer l'angle de dilatation à partir des essais. Mais cette quantification n'est pas toujours évident en raison de différents définitions ambiguës qui se présentent dans la littérature.

Cet article concerne l'interprétation des résultats de différents tipes d'essais de sols, principalement le procédure pour l'évaluation correcte de l'angle de dilatation. Quelques cas sont présentés et discutés.

Keywords : plasticity, dilatancy, soil testing, failure

1 INTRODUCTION

Dilatancy is an important aspect of soil behaviour. It manifests itself as a volumetric strain coupled to an applied shear strain. As discussed below, the angle of dilatancy, ψ, is a constant of a soil model, the elastic perfectly plastic Mohr-Coulomb (MC) model with a non-associated flow rule. It embodies the concept of dilatancy within the confines of the MC model. In real soils, dilatancy is variable and depends on soil density and stress level among other things (Kolymbas, 2000), while the MC model is only able to incorporate constant dilatancy, with the model's constant fixing the rate of dilatancy, ψ, being designated as the angle of dilatancy and measured in degrees. In other elastic perfectly plastic models, such as the Drucker-Prager model, there a material constant that plays the same role of fixing the rate of dilatancy but differs from the angle of dilatancy. The identification of the angle of dilatancy with the constant ψ in the MC model has two main advantages: the model is widely used by engineers in numerical analyses of geotechnical boundary value problems, and also the fact that this precise definition avoids ambiguities that arise when trying to compute values for the constant from soil tests.

The angle of dilatancy performs a similar role in defining the rate of volumetric strain to shear strain in the MC model, as the friction angle in fixing the rate of change of the shear strength with the effective mean stress. In real soils, the friction angle is variable like the angle of dilatancy. Also, dilatancy, for overconsolidated stress states above a certain magnitude, is connected with peak values of the friction angle that gradually decay with strain to critical state values. Dilatancy induces additional shear strength in the supercritical stress region, the implications of which, in the MC model context, are discussed in Schofield (2006).

The aim of this paper is to present a clear definition of the angle of dilation and how to obtain it from different types of soil tests.

2 DRAINED TRIAXIAL COMPRESSION TEST

The dilatancy angle is the constant of the Mohr-Coulomb (MC) model, ψ, that defines the plastic volumetric strain. Its role in the plastic potential function is analogous to the role of the friction angle, ϕ, in the yield function.

Traditionally, dilatancy in the conventional triaxial compression test is represented in a volumetric strain, ε_v, versus axial strain, ε_a, plot.

In what follows, the soil mechanics sign convention (compression positive) is adopted and stress stands for effective stress, despite the omission of the primes.

Under triaxial conditions, two principal stresses are equal, $\sigma_2=\sigma_3$, which implies that two mechanisms, defined by the following yield functions, are simultaneously active

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$$\begin{cases} f_1(\boldsymbol{\sigma}) = \sigma_1 - \sigma_3 - (\sigma_1 + \sigma_3)\sin\phi \\ f_2(\boldsymbol{\sigma}) = \sigma_1 - \sigma_2 - (\sigma_1 + \sigma_2)\sin\phi \end{cases}. \tag{1}$$

The first function applies when σ_3 is the minimum principal stress, while the second is valid when σ_2 is the minimum. σ_1 is the maximum principal stress in both cases. ϕ is the friction angle. The corresponding plastic potential functions, defining two plastic strain mechanisms are

$$\begin{cases} g_1(\boldsymbol{\sigma}) = \sigma_1 - \sigma_3 - (\sigma_1 + \sigma_3)\sin\psi \\ g_2(\boldsymbol{\sigma}) = \sigma_1 - \sigma_2 - (\sigma_1 + \sigma_2)\sin\psi \end{cases}, \tag{2}$$

where ψ is the dilatancy angle. When two principal stresses are equal, the stress is on an edge of the Mohr-Coulomb irregular pyramid. The flow rule includes the contribution of both plastic mechanisms

$$d\boldsymbol{\varepsilon}^p = d\gamma_1 \frac{\partial g_1}{\partial \boldsymbol{\sigma}} + d\gamma_2 \frac{\partial g_2}{\partial \boldsymbol{\sigma}}, \tag{3}$$

with two plastic multipliers defining the relative weight of each mechanism. As the stress state must satisfy simultaneously two yield conditions, it becomes possible to determine both plastic multipliers. In the deviatoric plane, the plastic strain increment direction can be located anywhere in the fan region limited by each mechanism as shown in Figure 1.

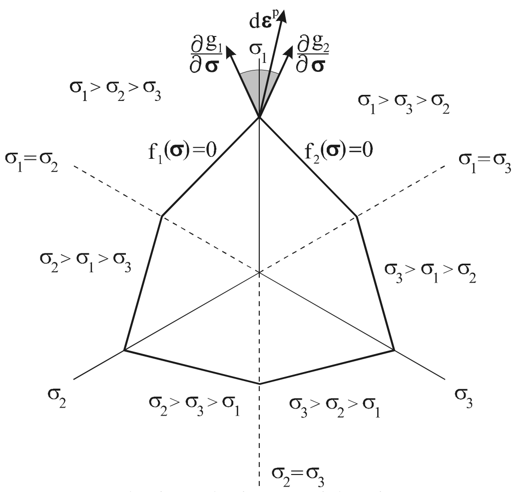

Figure 1. Two mechanism plastic potential acting at a MC triaxial compression edge, in the deviatoric plane.

In principal directions, the plastic potential functions stress gradients depend uniquely on the dilatancy angle

$$\left\{\frac{\partial g_1}{\partial \boldsymbol{\sigma}}\right\} = \begin{Bmatrix} 1-\text{sen}\,\psi \\ 0 \\ -1-\text{sen}\,\psi \end{Bmatrix} \quad \text{e} \quad \left\{\frac{\partial g_2}{\partial \boldsymbol{\sigma}}\right\} = \begin{Bmatrix} 1-\text{sen}\,\psi \\ -1-\text{sen}\,\psi \\ 0 \end{Bmatrix}. \tag{4}$$

The plastic strain increment will exhibit axial symmetry if the plastic multipliers are equal, that is, if the contributions of each mechanism are identical. Each mechanism essentially defines a plane strain state. In mechanism 1, the strain in direction 2 is zero, while in mechanism 2, the strain is zero in direction 3.

Assuming that, at failure, stresses remain constant and the elastic strains are negligible relative to the plastic ones, the rate of variation of the volumetric strain relative to the axial strain in the MC model is given by

$$\frac{d\varepsilon_v}{d\varepsilon_a} = \frac{d\varepsilon_v^p}{d\varepsilon_1^p} = \frac{(d\gamma_1 + d\gamma_2)(-2\sin\psi)}{(d\gamma_1 + d\gamma_2)(1-\sin\psi)} = \frac{-2\sin\psi}{1-\sin\psi}. \tag{5}$$

It is interesting to verify that the dilatancy rate does not depend on the actual values of the plastic multipliers, i.e., in the case of the MC model and under triaxial conditions, it does not depend on the direction of the plastic strain increment. The value of $d\varepsilon_v/d\varepsilon_a$ is obtained from the conventional triaxial test, taking into account that in the case of dilatancy (expansive volumetric strain), it is negative. The curves of volumetric strain versus axial strain relative to a conventional triaxial compression test on a dilatant soil (broken line) and resulting from the MC model (continuous line) are represented in Figure 2.

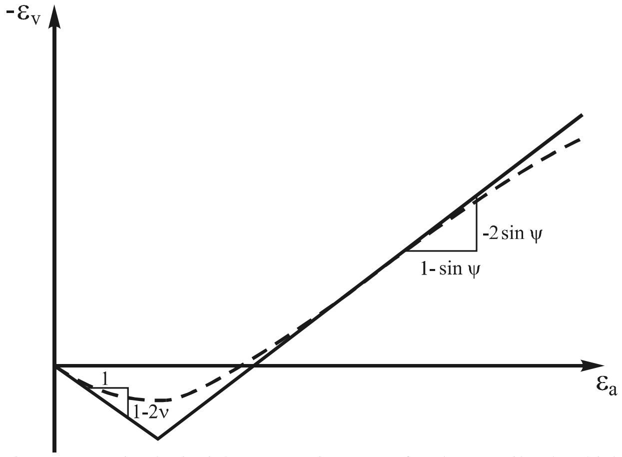

Figure 2. Drained triaxial compression test of a dense soil. The thick line is the MC model response.

Knowing the value of $d\varepsilon_v/d\varepsilon_a$ for a given triaxial test, the value of ψ is given by

$$\psi = \arcsin\left(\frac{\dfrac{d\varepsilon_v}{d\varepsilon_a}}{\dfrac{d\varepsilon_v}{d\varepsilon_a} - 2}\right). \tag{6}$$

This expression is the same as the one obtained by Vermeer (1984), even considering that they used the solid mechanics sign convention (compression negative).

3 DRAINED SIMPLE SHEAR TEST

In the drained simple shear test, the strain is less homogeneous then in the triaxial test, but more so than in the direct shear test (shear box). This test tries to reproduce simple shear conditions with vertical strain so that the sample can dilate or contract. The vertical stress is kept constant. This test is more difficult to analyse because there is rotation of the principal stresses. Also, plane strain conditions apply and only one plastic mechanism needs to be considered. In this case, assuming that σ3 is the intermediate principal stress, the yield function is f=f2 and the plastic potential is g=g2 as defined above. Using the spectral representation, the yield and plastic potential stress gradients are given by

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} = \sum_{i=1}^{3} \frac{\partial f}{\partial \sigma_i} \mathbf{m}_{(i)} \otimes \mathbf{m}_{(i)}, \quad \frac{\partial g}{\partial \boldsymbol{\sigma}} = \sum_{i=1}^{3} \frac{\partial g}{\partial \sigma_i} \mathbf{m}_{(i)} \otimes \mathbf{m}_{(i)}, \quad (7)$$

$$\text{and} \quad \boldsymbol{\sigma} = \sum_{i=1}^{3} \sigma_i \mathbf{m}_{(i)} \otimes \mathbf{m}_{(i)}$$

where σ_i are the principal stresses and $\mathbf{m}_{(i)}$ are the stress principal direction unit vectors. The principal direction vectors are assumed to be related to the reference basis {$\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$} by

$$\begin{cases} \mathbf{m}_{(1)} = \cos\theta \mathbf{e}_1 + \sin\theta \mathbf{e}_2 \\ \mathbf{m}_{(2)} = -\sin\theta \mathbf{e}_1 + \cos\theta \mathbf{e}_2 \\ \mathbf{m}_{(3)} = \mathbf{e}_3 \end{cases}, \quad (8)$$

where $\mathbf{e}_1$ is identified with the in-plane horizontal direction and $\mathbf{e}_2$ with the vertical one. θ is the angle that the major principal stress σ_1 makes with the horizontal (positive in the counter clockwise direction). In this test, only the vertical, ε_{22}, and shear, ε_{12}, strains are non zero.

From equations (4), (7) and (8) the following components from the plastic potential stress gradient are obtained

$$\frac{\partial g}{\partial \sigma_{11}} = \cos 2\theta - \sin\psi \quad , \quad (9)$$

$$\frac{\partial g}{\partial \sigma_{22}} = -\cos 2\theta - \sin\psi \quad \text{and} \quad \frac{\partial g}{\partial \sigma_{12}} = \sin 2\theta \cdot \quad (10)$$

At failure, a steady state is reached and stresses don't change anymore, such that

$$d\boldsymbol{\sigma} = \mathbf{D} : d\boldsymbol{\varepsilon}^e = 0 \Rightarrow d\boldsymbol{\varepsilon}^e = 0 \Rightarrow d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^p = d\gamma \frac{\partial g}{\partial \boldsymbol{\sigma}}, \quad (11)$$

thus the elastic strain increments vanish and the strain increments become completely plastic. Because the horizontal strain must be zero

$$d\varepsilon_{11} = d\varepsilon_{11}^p = d\gamma \frac{\partial g}{\partial \sigma_{11}} = 0 \Rightarrow \cos 2\theta = \sin\psi \quad . \quad (12)$$
$$\Rightarrow \sin 2\theta = \cos\psi$$

From equations (12) and (10) the final simple result is obtained

$$\frac{d\varepsilon_{22}}{d\gamma_{12}} = \frac{d\varepsilon_{22}^p}{2d\varepsilon_{12}^p} = -\frac{\sin\psi + \sin\psi}{2\cos\psi} = -\tan\psi \cdot \quad (13)$$

This result confirms that, in the simple shear test, at failure, the tangent of the dilatancy angle is the ratio of the vertical stress increment to the shear strain increment. This is the test where the dilatancy angle can most readily be identified.

4 DRAINED BIAXIAL PLANE STRAIN TEST

In the biaxial plane strain test, the principal stresses cannot rotate and the strain state in the sample is more homogeneous than in the simple shear test. At failure, considering, as in the case of simple shear, that σ_3 is the intermediate (out of plane) principal stress, the principal strain ratio is

$$\frac{d\varepsilon_1}{d\varepsilon_2} = \frac{d\varepsilon_1^p}{d\varepsilon_2^p} = \frac{d\gamma \frac{\partial g}{\partial \sigma_1}}{d\gamma \frac{\partial g}{\partial \sigma_2}} = \frac{1-\sin\psi}{-1-\sin\psi} \cdot \quad (14)$$

solving for sin ψ, results in

$$\sin\psi = -\frac{d\varepsilon_1^p + d\varepsilon_2^p}{d\varepsilon_1^p - d\varepsilon_2^p} \cdot \quad (15)$$

This result is the same as the one given in Roscoe (1970), but with the sign changed due to a different sign convention.

5 DILATANCY RATE

A possible definition for the rate of dilatancy in elastoplastic models, applicable to any type of soil test is the one given by De Simone and Tamagnini (2005), that define it as the ratio, d, between volumetric and deviatoric plastic strain rates,

$$d = \frac{d\varepsilon_v^p}{d\varepsilon_s^p}, \quad d\varepsilon_v^p = \text{tr}(d\boldsymbol{\varepsilon}^p)$$
$$d\varepsilon_s^p = \sqrt{\frac{2}{3} d\mathbf{e}^p : d\mathbf{e}^p}, \quad d\mathbf{e}^p = \text{dev}(d\boldsymbol{\varepsilon}^p). \quad (16)$$

Unfortunately, this definition of the rate of dilatancy, is not uniquely related, at failure, to the angle of dilatancy for all soil test types, as can be deduced from the cases presented above.

6 DILATANCY AND UNDRAINED CONDITIONS

When soil loading takes place under undrained conditions, the assumption of zero volumetric strain is valid if both the fluid and soil particles are considered incompressible. For an isotropic linear elastic perfectly plastic soil model this implies

$$dp' = K' d\varepsilon_V^e = -K' d\varepsilon_V^p = -d\gamma K' \text{tr}\left(\frac{\partial g}{\partial \boldsymbol{\sigma}'}\right), \quad (17)$$

where K' is soil matrix bulk modulus.

In the case of the MC model, the mean effective stress increment at failure is given by

$$dp' = -K' d\varepsilon_V^p = 2 d\gamma K' \sin\psi \cdot \quad (18)$$

This means that, under failure conditions, positive dilatancy (ψ>0) gives rise to an increase in the mean effective stress with the associated increase in shear strength. The implication is that the material will not fail. The undrained shear strength will be infinite unless a provision is taken to limit the pore pressure to minus 100kPa, the point at which cavitation will occur. If the MC model is being used in finite element analysis of problems involving stability and undrained conditions, the dilatancy angle should be taken as zero.

7 TRIAXIAL COMPRESSION EXAMPLE

In order to clarify what has been described above, a practical example of the determination of the friction angle in the case of a drained triaxial compression test (Portugal, 1999) is presented. The tested soil is a Fontainebleau sand sample. It is a fine sand, with almost constant grain size and 89% relative density. The applied cell pressure is 200kPa. The deviatoric stress presents a peak at an axial strain value between 4% and 5% (see Figure 3). The post peak stress deviatoric stress reduction is approximately 15% of the peak value. The peak friction angle is 43º.

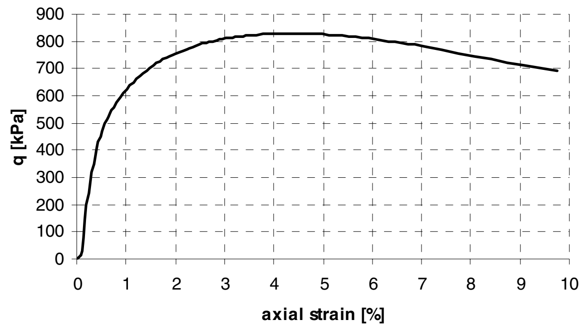

Figure 3. Deviatoric stress vs. axial strain in triaxial test.

The measured volumetric strain is shown in Figure 4 as a thick line. It must be mentioned that here, in contrast to common practice, the dilatant volumetric strains are negative in order to be consistent with the soil mechanics convention that considers compressive stresses to be positive. In the first stage of the test, below 0.5% axial strain, there is negative dilatancy, because, at that stage, the behaviour is mainly elastic, and that is the elastic response to the mean stress increase. In the next stage, the gradual onset of dilatancy originates an increase in the rate of dilatancy until a maximum value is attained at the same axial strain as the peak deviatoric stress. In the final stage, the rate of dilatancy starts decreasing and will tend to zero as the critical density is approached.

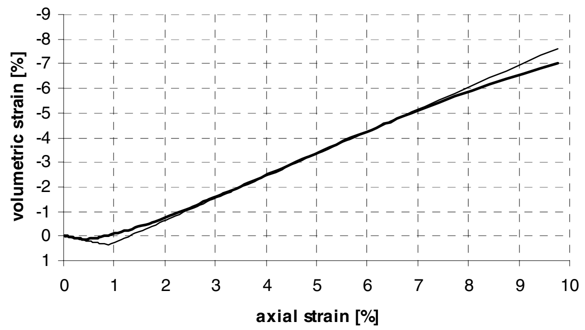

Figure 4. Volumetric strain vs. axial strain in triaxial test. Measured (thick line). MC model (thin line).

The rate of dilatancy, here measured as the ratio between the volumetric and the axial strain increments, is plotted versus the axial strain in Figure 5. There are jumps in the computed dilatancy rate from increment to increment, so in order to present a clearer trend, a smoothed curve, obtained by a moving average procedure is represented as a thick line. It can be confirmed, from the smoothed curve, that the maximum rate of dilatancy is attained at an axial strain value between 4% and 5%, coinciding with the deviatoric stress peak. The maximum value of the dilatancy rate is about -0.9. Using equation (6), the value of the dilatancy angle, ψ, is computed as 18º. The resulting MC volumetric strain curve is then represented in Figure 4 as a thin line. It can be seen that, in this case, the MC line approximates quite well the measured curve over a wide range of axial strain values.

The strength and dilatancy rate reduction that takes place in dense soils must be considered when dealing with problems involving stability. When using the MC model to evaluate the stability of a geotechnical structure it is preferable to make ψ=0.

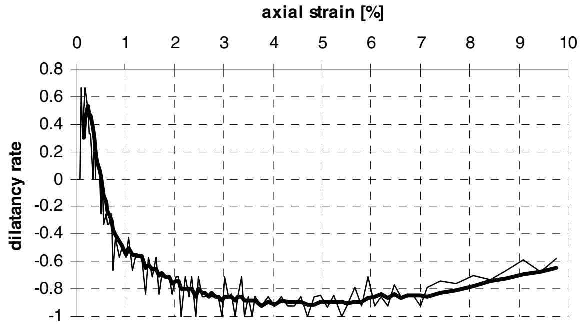

Figure 5. Dilatancy rate vs. axial strain in triaxial test. The thick line is a moving average.

It must always be taken into account that dilatancy is invariably associated, in all test types, with strain localization in the sample immediately after, or even before, the deviatoric stress peak is attained. As the strain and stress states are computed assuming that they are homogeneous in the sample, the real local material response might differ quantitatively.

8 CONCLUSIONS

The angle of dilatancy is a constant of the MC elastic perfectly plastic model with a non-associated flow rule where the plastic potential function has the same form of the yield function. This is a unique and precise definition.

Explicit expressions for the angle of dilatancy at failure where then derived for the drained triaxial compression, simple shear and plane strain biaxial tests.

A practical example of the evaluation of the dilatancy angle in the case of a drained triaxial compression test on a dense sand was presented.

It was also pointed out, that the use of a positive dilatancy angle for the MC model in undrained conditions, has the serious implication that the soil will not fail.

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