Probabilistic methods in the stability analysis of earth retaining structures

La Méthode Probable de l'analyse de la Stabilité de Structures Retenant la Terre

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ABSTRACT

The uncertainty and variability associated with soil parameters are conventionally accounted for in practice by the adoption of point estimates of parameters, with the estimated values reflecting the engineer's confidence level in the observed data. This approach is rather simplistic and falls short of providing enough information on the bulk of available data. The statistical and probabilistic method of analysis is a rational and systematic approach that recognizes the variability of soil properties and provides reliable estimates of soil parameters for design purposes. For homogeneous and slightly heterogeneous soils, the mean value of the parameters from probabilistic analysis compares favorably with single value estimates obtained from conventional soil analysis. Reliability assessment of the stability of a retaining wall located in a project area of slightly heterogeneous soils in the Niger Delta region has been carried out using rigorous analytical methods and Microsoft – Excel spreadsheet optimization. The two approaches produced fairly similar results for parameters obtained from conventional and probabilistic soil analysis. On this basis, it is therefore affirmed that parameters derived from conventional analysis of samples of homogenous and slightly heterogeneous soils are adequate for design. Hence, there is no need to embark on the rigorous and complex probabilistic analysis, particularly in projects of moderate scale. This may only be necessary in large scale projects and in sites where soils exhibit pronounced heterogeneity. The spreadsheet-based reliability analysis however exhibits versatility and is recommended as a convenient analytical tool in the stability analysis of retaining walls. Its ability to explicitly reflect the correlation, standard deviation, probability distributions and sensitivities and to automatically seek the most probable failure combination of parametric values for any case under consideration gives it an edge over other methods of analysis.

RESUME

La variabilité et incertitude associent aux paramètre de la terre sont conventionnellement justifie dans le pratique par l'adoption des points de devis des paramètres auquel les valeur reflets les niveaux de confiance d'un ingénieur a des données observée. Cette approche sont plutôt très simple mais elle ne donne pas un renseignement sufficit pour un ensemble des données disponible. La méthode d'analyse par probabilité et statistique sont des approche rationnels et systématique qui reconnaisse des propriété variable et donne des devis des paramètre de terre plus acceptable en dessin. Pour la terre un peu heterogene et homogène les valeurs moyen des analyse des paramètre probabilité se compare bien avec les valeur des devis obtenu des analyses de la terre conventionnel. L'essaye d'endurance de la stabilité d'un murs, situe dans la région des projets de terre un peu heterogene au Niger Delta, a été fait par la méthode d'analyse rigoureux et optimisation de Microsoft 'spread-sheet'. Les approches produit quelque peu des résultats similaires a des paramètre obtenu des analyse de la terre conventionnelle et avec de probabilité. Donc en peux conclu que les analyses des paramètres conventionnel sont adéquate pour être utilise en dessin de terre un peu heterogene et homogène et il n'y a pas l'avantage important pour embarque a des analyses de probabilité rigoureux et inutile. L analyse probabilité ne qu'utile dans les projets de grand chantier et dans les situation ou les samplers de terre heterogene sont évident. L'analyse de surette base sur le 'spread sheet' démontre quelque versatilité et est donc recommande comme l'utile le plus convenaient pour des analyse de stabilité pour les murs retenant. Sont abilite de bien réfléchir le corrélation, la déviation standard, la sensitivité et probabilité distribution et automatiquement cherche des valeurs paramétriques de combination le plus ruiner de tous case qui doit être considérer lui donne une écart sur des méthodes des analyses.

Keywords: Uncertainty; Statistical analysis; Stability analysis; Probability; Retaining wall; Microsoft-excel

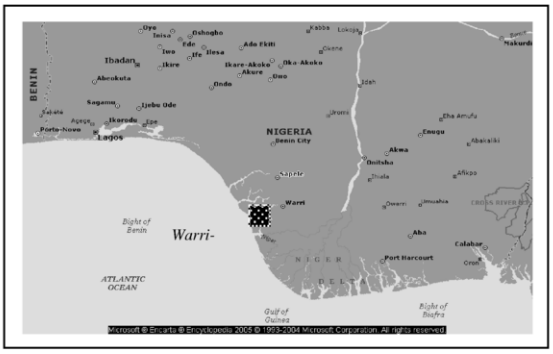

Figure 1. Map of Niger Delta Region-Southern Nigeria

1 INTRODUCTION

In reality, soil is spatially variable and its properties exhibit some degree of variability going by the spectra of values observed for a soil property in repeated laboratory tests conducted under similar conditions. The variability in soil properties is attributed to changes related to depositional and post-depositional history and processes.

Traditional design of retaining wall assumes soil to be spatially uniform leading to the adoption of single value estimates for soil parameters. Single value estimates for soil properties are sometimes obtained by taking the average of the maximum and minimum values of a set of data from laboratory test results. This method does not convey enough information on the bulk of data but places emphasis on the limits in the range of data. Such a simplistic approach could provide unrealistic values especially where the occurrence of the extreme values is as a result of errors occasioned by measurement and engineering defects and where heterogeneity is pronounced within the soil.

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The need to develop a systematic and rational approach of evaluating results of laboratory test that will address the variability in soil properties and provide reliable estimates of soil parameters even with the few samples usually available in practice has been stressed (McGuffey et al, 1981). Work on soil property variability, using statistical and probabilistic concepts, has progressed over the years (e.g. Matsuo, 1976; Harr, 1977; Lumb, 1975). The use of such concepts allows one to make inference decisions about the properties of a specific soil deposit.

In practice, the method involves the determination of appropriate probability distributions which best model the soil parameters of interest. In many practical situations, the amount of information available is often limited and this does not allow for the theoretical development of a probability distribution which truly represents the soil parameter. In realization of this limitation, approximate distribution functions are sometimes predicted and estimates of population parameters made within confidence limits. In other situations, histograms of the data obtained for the various soil properties are plotted and an indication of the type of distributions they can assume is obtained using their statistical descriptions and Pearson chart. For higher reliability, standardized distribution curves can be fitted into the histogram and the one that gives the best fit represents the probability density function. Computer soft-wares are readily available for more accurate prediction of distributions (e.g. @ Risk).

This paper presents a probabilistic analysis of soil encountered at a project site in the Niger Delta region of Southern Nigeria and describes the probabilistic stability analysis of a retaining wall based, in the first instance, on an analytical method and then by spreadsheet optimization of the Hasofer-Lind reliability index (Low & Phoon, 2002). Results from the two computations are compared. The study also applied the analytical method to evaluate the reliability of the stability assessment of a retaining wall made using data from conventional soil analysis. The usefulness or otherwise of a rigorous probabilistic analysis in soils of varying degree of homogeneity is highlighted based on the results of the computations.

2 THEORETICAL OVERVIEW

2.1 *Soil Property Variability and Distribution functions*

Engineering properties of soil exhibit considerable degree of variability from one point to another even when the soil layers are considered homogeneous (Vanmarcke, 1977). The variation in soil properties follows a given trend when studied in three dimensions, being more pronounced in the vertical direction than in the horizontal direction. This trend is attributed to the state of stress which varies with depth and the varying age of deposits over time with layers at different ages of digenesis.

Soil properties are random variables with defined probability density functions (McGuffey et al, 1981). Earlier studies identified the statistical distributions of many soil properties as conforming to the theoretical normal distribution Matsuo (1976). In reality however, the normal distribution model does not adequately represent the distributional characteristics observed for most soil properties. The distributions for most soil properties exhibit certain degree of skewness in difference to the symmetric shape of a normal distribution. Beta, Log-normal, Gamma distributions are more appropriate models for most soil properties (Harr, 1977). The random variables in the stability analysis of a retaining wall include unit weight, angle of internal friction and cohesion and they occur frequently in expressions for earth pressures. Phoon (2003) presents a statistical guideline for soil property evaluation.

2.2 *Performance Function*

In general form, the problem requiring probabilistic analysis is expressed as a vector **X**=[X_1, X_2, X_3... Xn] representing a set of random variables in which a limiting state or performance function given by

$$g\left(X_1, X_2, X_3 ..., X_n\right) = 0, \tag{1}$$

defines a n-dimensional critical hyper-surface, such that g(.)>0 is the safe state and g(.)<0 is the failure state. The probability of failure is then given by the following integral:

$$P_f = P\left[g\left(\mathrm{X} \leq 0\right)\right] = \int_{g(\mathrm{X})\leq 0} f\left(\mathrm{X}\right) d\mathrm{X} \tag{2}$$

where $f(\mathrm{X})$ is the joint probability function with the integral performed over the failure domain. Computations to define the failure points located on the hyper-surface provide a measure of the reliability index (β).

2.3 *Determination of Reliability Index*

The minimum distance from the critical hyper-surface to the origin is taken as a measure of the reliability index as this distance defines the most probable failure point (Shinozuka, 1983). Using Lagrange multipliers, the failure point can be obtained by minimizing the function,

$$\left(X_1^2 + X_2^2 + \cdots + X_n^2\right)^{1/2}$$

$$g\left(X_1, X_2, X_3, ..., X_n\right) = 0$$

subject to the constraint, For a performance function that is non-linear solution, it is difficult and cumbersome to compute the exact failure point. An approximate solution is obtained by assuming a hyper-plane tangent to the hyper-surface as the failure plane. The *i*th component of the failure point, $\left(x_{1f}^{'}, x_{2f}^{'}, ..., x_{nf}^{'}\right)$, expressed in reduced variates is determined from the expression:

$$x_{if}^{i} = \frac{\left(\frac{\partial g}{\partial X_i^{'}}\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i^{'}}\right)_f^2}}\beta = -\alpha_i \beta, \tag{3}$$

where α_i are direction cosines. With the derivatives performed at points defined in reduced variates, the real failure points are computed from the expression:

$$x_{if} = \mu_i + \sigma_i x_{if}^{'} = \mu_i - \alpha_i \beta \tag{4}$$

Substituting for x_i in equation (1) will provide a value for the reliability index.

The probability of failure will then be given as

$$P_f = P\left[g\left(\mathrm{X}\right) \leq 0\right] \approx \Phi\left(-\beta\right) \tag{5}$$

Low and Tang (1997) present an interpretation of the Hasofer-Lind reliability index using Microsoft excel spreadsheet optimization. The approach evaluates the reliability index taking into consideration the mean values of the parameters and their scatters.

The Matrix formulation (Ditlevsen, 1981) of the H-L index is given by:

$$\beta = \min_{\underline{x} \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T \left[\underline{R}\right]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \tag{6}$$

Details of the practical procedure was presented in Low & Tang (1997), and some other practical applications in Low & Phoon (2002).

3 PROBABILISTIC STABILITY ANALYSIS OF CANTILEVER RETAINING WALL

3.1 *Probabilistic Evaluation of Soil Property*

The project site adopted for the study is underlain by silty sand and soft silty clay. Laboratory tests were carried out on limited number of samples (<30). The description statistics of data from laboratory tests were used to predict the probability distribution function of the relevant soil properties. The predicted models for the soil properties are as in Table 1.

Table 1. Predicted Probability Distribution Function

Soil Type	Soil Properties		
	Unit wt (*γ*)	Friction angle(*φ*)	Cohesion (*c*)
Silty Sand	Normal dist.	Beta dist.	
Silty Clay	Normal dist.	Beta dist.	Normal dist

.

Given the small number of samples drawn and the limited data available for analysis, the population standard deviation and population mean are unknown and need to be estimated. The domain of occurrence of the expected value is determined using the t-statistics for desired confidence limits. The relevant description statistics is given in Table 2 while the lower and upper limits for the soil properties for 95% confidence level are as in Table 3.

Table 2. Relevant Description Statistics

Soil Type	Soil Property	Sample mean	Std Deviation	COV
Silty Sand	γ_1	18.27	0.44	0.024
	φ_1	29.70	1.92	0.065
Silty Clay	γ_2	12.97	1.45	0.11
	φ_2	4.13	1.10	0.27
	c	23.78	7.69	0.32

Table 3. Confidence Intervals at 95% confidence limits

Soil Type	Soil Property	Lower limit	Upper limit
Silty Sand	γ_1	18.08	18.46
	φ_1	28.88	30.52
Silty Clay	γ_2	12.34	13.59
	φ_2	3.65	4.61
	c	20.46	27.11

Table 4. presents the recommended design values for the soil properties using conventional soil analysis.

Table 4. Design values from Conventional Soil Analysis

Soil Type	Soil Property	Range		Rec. Value
		Max	Min	
Silty Sand	γ_1	17.57	19.25	18.41
	φ_1	27.00	33.00	30.00
Silty Clay	γ_2	11.06	16.11	13.59
	φ_2	2.00	6.00	4.00
	c	10.00	41.00	25.50

3.2 *Retaining Wall Performance Function*

For a retaining wall, three modes of failure are identified, namely rotation about the toe of the wall (overturning), horizontal sliding along the base of the wall and bearing capacity failure of the soil beneath the wall. Considering the cantilever retaining wall (fig 1), the performance functions (PerFn1, PerFn2, PerFn3) with respect to rotation (overturning), sliding and bearing capacity modes are, respectively given as:

$$PerFn1 = W_1 Arm_1 + W_2 Arm_2 + W_3 Arm_3 + W_4 Arm_4 - P_a Arm = 0 \quad (7)$$

$$PerFn2 = \Sigma V \tan\phi_2 + Bc + P_p - P_a = 0 \quad (8)$$

$$PerFn3 = q_u - q = 0 \quad (9)$$

Where W_1, W_2, W_3 represent the component weights of the retaining wall and Arm_1, Arm_2, and Arm_3 are their corresponding lever arms about the toe (Z), W_4 is the weight of retained soil on the cantilever and Arm_4 is its lever arm about Z. $\Sigma V = W_1 + W_2 + W_3 + W_4$.

Using simple regression analysis, the performance functions expressed in terms of the variable soil properties are modeled as:

$$PerFn1 = 452 + 8.4\gamma_1 - \gamma_1\phi_1 = 0 \quad (10)$$

$$PerFn2 = 2.5\gamma_1\phi_1 - 3.25\gamma_1\phi_2 - 3.1\gamma_2\phi_2 - 12.5\phi_2 c - 1146 = 0 \quad (11)$$

$$PerFn3 = 2\gamma_1 - 2\phi_1\gamma_2 - \phi_2 c + 473 = 0 \quad (12)$$

For b=0.3m, d=0.4m, e=1.0m, f=0.2m B=4m, H=5.2m, D=1.0m

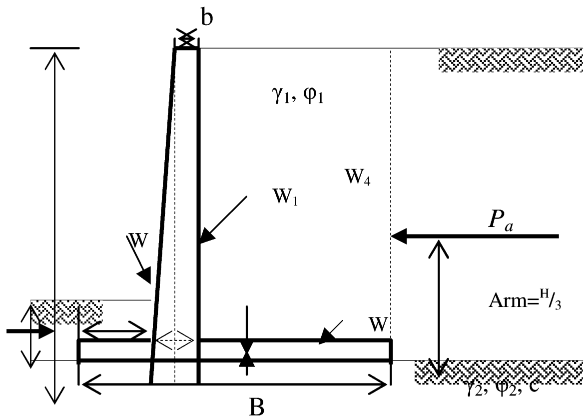

Figure 1. Cantilever Retaining Wall

3.3 *Results from Analysis*

The analytical method is an iterative approach of determining the failure points on the hyper-plane using equations (3) and (4) constrained by equations (10), (11) and (12) when dealing with respective modes of failure. The first iteration is performed adopting the sample mean and standard deviation as initial values. Results from analysis are presented in Table 5.

Table 5. Results of Reliability Analysis of Retaining wall (Analytical)

Mode of Failure	Reliability index β	Reliability r=Φ(β)	Probability of failure P_f=1-Φ(β)
Overturning	2.105	0.9821	0.0179
Sliding	1.590	0.9440	0.0392
Bearing Capacity	1.709	0.9563	0.0437

Table 6: Results of Reliability Analysis of Retaining wall Using recommended value from conv. Soil analysis (Analytical)

Table 6. Results of Reliability Analysis of Retaining wall Using recommended value from conv. Soil analysis (Analytical)

Mode of Failure	Reliability index β	Reliability r=Φ(β)	Probability of failure P_f=1-Φ(β)
Overturning	1.76	0.96080	0.0392
Sliding	1.68042	0.95352	0.04648
Bearing Capacity	1.9568	0.97500	0.02500

Table 7. Results of Reliability Analysis of Retaining wall(Excel Spreadsheet optimization of H-L index)

Mode of Failure	Reliability index β	Reliability r=Φ(β)	Probability of failure P_f=1-Φ(β)
Overturning	1.915	0.9723	0.0277
Sliding	1.736	0.9587	0.0413
Bearing Capacity	2.833	0.9977	0.0023

The failure mode of a retaining wall is modeled as a series system since failure by any of the modes will lead to failure of the entire structure. The reliability and probability of failure of the retaining wall, based on the results from reliability analyses for the different modes of failure is given in Table 8

Table 8. Overall Reliability of Retaining Wall

Method of Analysis	Reliability $r = \prod_{i=1}^{n} 1 - p_i$	Prob. of Failure (%)
Analytical	0.902384	9.76
Excel Spreadsheet	0.92997	7.00
Analytical (conv. Data)	0.8932	10.68

4 SUMMARY AND CONCLUSION

On the basis of results obtained from laboratory tests and analysis, the following observations can be made:

a) The soil properties exhibit varying degree of variability. Variability is more pronounced in the silty clay than in the silty sand. This is evident in low COV values for γ and φ in silty sand indicating low variability and high COV for the strength parameters in silty clay indicating high variability.

b) The recommended values from conventional soil analysis fall within the limits established by probabilistic analysis at 95% confidence limit. This observation and the range of COV for the parameters, suggest that the heterogeneity of the soil investigated is not pronounced.

c) Evaluation of the reliability of a retaining wall by analytical method and spreadsheet optimization gave probability of failure of 9.76% and 7% respectively while that based on the recommended design values from conventional analysis is 10.68%.

In summary, the use of probabilistic soil analysis produced no clear advantage over the conventional soil analysis for a soil considered homogeneous and slightly heterogeneous. For such soils, the conventional methods of soil analysis will suffice. Probabilistic approach will be realistically advocated where soils of pronounced heterogeneity are encountered and in large scale projects where the stakes are quite high.

Results of the Analytical and the Spreadsheet-based reliability analysis show close agreement The Spreadsheet-based reliability analysis is versatile and can be conveniently applied in the analysis of a retaining wall

REFERENCES

Ditlevsen, O. (1981). Uncertainty Modeling: with Applications to Multidimensional Civil engineering Systems. New York: McGraw-Hill.

Harr, M. E. (1977), Mechanics of Particulate Media – A Probabilistic Approach, McGraw-Hill Publishers New York.

Hasofer, A.M. and Lind, N.C. (1974). "Exact and Invariant Second-Moment Code Format." *J. Engng Mech. 100*, 111-121.

Low, B.K and Phoon, K.K. (2002) "Practical First-order Reliability Computations using Spreadsheet." *Proceedings of the Conference on Probabilistic in Geotechnics: Technical and Economic Risk Estimations*, pp. 39-46. Essen: Verlag Glückauf GmbH

Low, B.K. and Tang, W.H. (1997a) **"**Efficient Reliability Evaluation Using Spreadsheet.**"** *J. Engng Mech., ASCE 123, No. 7*, 749-752.

Lumb P (1975), "Spatial Variability of Soil Properties" *Application of Statistics and Probability in Soil*, *Proc. of International Conference,* Sacher, Feb-Sept 1975

Matsuo M. (1976) "Statistical Properties of Undrained Strength of Saturated Clay" *Risk and Decisions in Geotechnical Engineering,* Vanmercke E.H. ed., MIT Special Summer Project, June 28- July 2, 1976.

Lumb P (1975), "Spatial Variability of Soil Properties" *Application of Statistics and Probability in Soil*, *Proc. of International Conference,* Sacher, Feb-Sept 1975

McGuffey, V.; Iori, J.; Kyfor Z and Grivas D.A., (1981) "Statistical Geotechnical Properties of Lockport Clays" *Paper prepared for the Annual meeting of the Transportation Research Board,* Washington D.C., Jan. 1982.

Shinozuka, N. (1983), "Basic Analysis of 109, pp721-740.

Vanmarcke, E.H. (1977), "Probabilistic Modeling of Soil Profiles" *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 103, No GT11 Nov 1977.