Probabilistic liquefied shear strength criteria from case histories

Critères probabilistes de résistance liquéfiés au cisaillement des antécédents

M. Gutierrez
Colorado School of Mines, Golden, CO, U.S.A.

M. Eddy
Virginia Tech, Blacksburg, VA, U.S.A.

ABSTRACT

Liquefaction of granular soils can have extremely detrimental effects on the stability of soil slopes and deposits, and on structures founded on them. A critical parameter in the evaluation of the liquefaction of soils is the residual or liquefied shear strength. This liquefied shear strength determines the magnitude of the deformation that the soil will undergo once it has liquefied. Current procedures for estimating the liquefied shear strength are based on laboratory testing programs, or from the back-analysis of case histories of liquefaction failures where in-situ test data were available. The case-histories approach is the procedure that is preferred in practice. However, it has several limitations including the very limited amount of data available, the significant uncertainties involved in the back-calculation of the liquefied shear strengths, and the lack of consistent and rational methods in the use of the available data. To address these current limitations, this paper proposes new probabilistic liquefied shear strength criteria for liquefiable soils from case histories.

RÉSUMÉ

La Liquéfaction des sols granulaires peut avoir des effets extrêmement préjudiciables sur la stabilité du sol des pentes et des dépôts, et sur les structures fondées sur eux. Un paramètre critique dans l'évaluation de la liquéfaction des sols est le liquide résiduel ou resistance au cisaillement du liquide. Cette resistance au cisaillement du liquide détermine l'ampleur de la déformation dont le sol fera l'objet une fois qu'il soit liquéfié. Les procédures actuelles d'estimation de la résistance au cisaillement du sol liquéfié sont fondées sur les programmes d'essais en laboratoire, ou sur l'analyse hystorique des cas où les données des tests échoués de liquéfaction au sous-sol sont disponibles. Etudier l'historique des cas est la procédure qui est privilégiée dans la pratique. Toutefois, elle présente plusieurs limites, y compris la très faible disponibilité de données, les incertitudes importantes dans le calcul de la resistance au cisaillement du sol liquéfié, et l'absence de méthodes cohérentes et rationnelles à l'utilisation des données disponibles. Pour répondre à ces limitations actuelles, ce document propose un nouveau critère probabiliste de resistance au cisaillement du sol liquéfié pour des sols liquefiables.

Keywords : liquefaction, cohesionless soils, undrained shear strength, probabilistic, case histories, in situ tests

1 INTRODUCTION

According to Seed (1987), the two important aspects related to the liquefaction of soils are: 1) the stress conditions that trigger liquefaction, and 2) the consequences of the liquefaction. The first one requires the determination of the liquefaction shear strength, and the second one the post-liquefaction shear strength. It is now increasingly being recognized that the determination of the undrained residual shear strength could be more important than the determination of the stress conditions that trigger the liquefaction itself (e.g., Ishihara 1993; Stark et al. 1997; Finn 2000). The undrained residual or liquefied shear strength is the main factor which controls whether flow failure or large deformations will occur. As pointed out by Seed (1987), it may be adequate and economically advantageous simply to ensure the stability of an earth deposit or structure against post-liquefaction failure after the strength loss has been triggered than to prevent the triggering itself.

There are currently two methods for estimating the residual strength of soil deposits. One method is the case histories approach where the liquefied shear strength is back calculated from known cases of liquefaction in soil zones where in situ test data (e.g., Standard Penetration Tests results) were available. The other approach for determining the residual shear strength is the laboratory procedure. Poulos et al. (1985) have developed a procedure for flow liquefaction using the results of monotonically loaded, consolidated-undrained triaxial tests. Ideally, the residual shear strength should be determined using undisturbed samples. However, it is often difficult to obtain high-quality undisturbed soil samples needed to determine the residual shear strength. More importantly, the costs of sampling and laboratory testing required in the laboratory approach are generally prohibitive, making the procedure applicable only for critical and large projects.

Although the use of field data and case histories should be preferred in practice, there are several limitations of the case-histories approach. These are: 1) limited amount of data on back-calculated residual shear strengths from field case histories, 2) significant uncertainties involved in the back-calculation of the residual shear strengths, and 3) lack of consistent and rational methods to use the available data on residual shear strength of granular soils. In order to address the current limitations in evaluating the liquefied shear strength of cohesionless soil deposits using in situ tests, this paper aims to: 1) re-evaluate and expand the available database on residual shear strengths of liquefied soils, 2) clearly delineate and systematically analyze the magnitudes of uncertainties involved in evaluating post-liquefaction shear strength, 3) develop robust, reliability-based procedures for back-calculating residual shear strength from case histories, and 4) present new probabilistic liquefied shear strength criteria for post-liquefaction stability analysis of cohesionless soils.

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2 CASE HISTORIES AND METHOD OF BACK-ANALYSIS

Thirty-eight case histories of flow liquefaction failures of natural and engineered slopes, including earth dams and embankments, were analyzed to obtain data on liquefied shear strength of cohesionless soils. The case histories are composed of 18 failures which were analyzed with the infinite slope model and 20 cases which were analyzed using the more general Spencer's (1967) method of slices. Only case histories with sufficiently good quality data to perform the probabilistic back-analysis were included. The case histories include re-analyses of 29 cases that were included in the study performed by Olson & Stark (2003), and nine new cases from recent earthquakes, including the 1993 Kushiro-oki Earthquake, 1993 Hokkaido-Nansei-oki Earthquake, 1995 Hyogoken-Nambu Earthquake, and the 1999 Kocaeli/Izmit Earthquake. The re-analyses differ from the analyses done by Olson & Stark (2003) in the following aspects: (1) the re-analyses did not account for kinetic forces, (2) the minimum, instead of the average value, of $(N_1)_{60}$ was used, (3) the SPT blow counts were not corrected for fines content, and (4) the liquefied shear strengths were not normalized with respect to the initial effective vertical stress.

The post-failure geometries were used for the back-analyses since the liquefied shear strength is mobilized in conjunction with the post-failure geometry (i.e., after liquefaction has been triggered). The undrained shear strength in each field case was systematically adjusted until the slope stability model matches the observed field post-failure geometry. Best estimates of parameters required in the slope stability analyses were used.

The slip surface for the infinite slope cases was defined by the depth to the water table and the height of water above the failure surface. These values are assessed based on available cross-sections and measurements of the likely zone of liquefied material. For the analysis of the complex cases, the slip surfaces for back-calculating the liquefied shear strength are determined by finding the minimum factor of safety surface corresponding to the liquefied shear strength resulting in a factor of safety of unity. This provides a consistent means for selecting a slip surface for analysis as the slip surface in the field does not necessarily correspond to limit equilibrium slip surface.

Figure 1 shows the back-analyzed undrained liquefied shear strengths S_{u_LIQ} vs. $\min(N_1)_{60}$ obtained from the 38 case histories. It was found that the best correlation between S_{u_LIQ} and $\min(N_1)_{60}$ is obtained when the SPT blow count is not corrected for fines content. Also, as discussed by Fear & McRoberts (1995), and Wride et al. (1999), the use of average SPT blow counts typically provides conservative values of liquefied shear strength. They proposed the use of the minimum SPT blow count as the "weakest-link-in-the-chain" measure. The $\min(N_1)_{60}$ values turn out to correlate well with the undrained shear strength that leads to liquefaction flow failure.

Different types of regression were tried to develop the best approximation of the back-analyzed liquefied shear strengths S_{u_LIQ} vs. the SPT blow count $\min(N_1)_{60}$ data obtained from the case histories. The correlations included: linear, power, logarithmic, exponential, and second order polynomial equations. It was found that the second-order polynomial shown in Figure 1 provides the best-fit. Figure 1 also shows the corresponding to plus and minus one standard error of estimate. The R^2-value for the best fit curve is about 0.54. Approximately 71% (27 of 38 cases) of the case histories fall within the one standard error of best fit curve. In conventional analysis, the best fit line can be used to provide the best estimate of S_{u_LIQ} from the measured value of $\min(N_1)_{60}$.

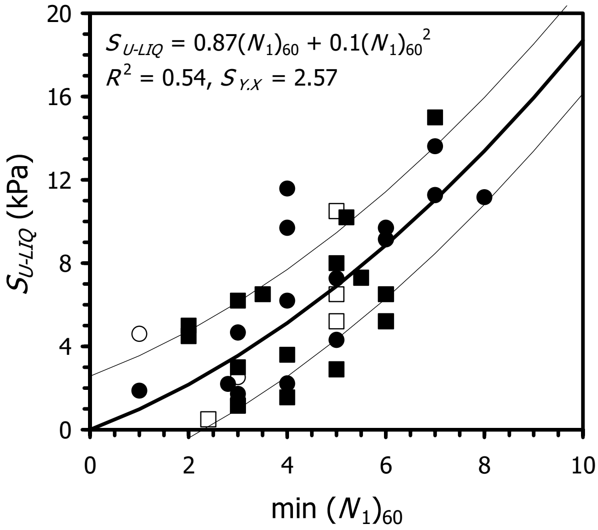

Figure 1. Relationship between liquefied shear strength and SPT blow count from case histories of flow liquefaction. Squares correspond to cases analyzed with Spencer's method and circles correspond to infinite slope cases. Solid symbols indicate that SPT data was measured at the site and open symbols indicate that SPT data was estimated.

3 PROBABILISTIC PROCEDURES

The First-Order Reliability Method (FORM) and Monte Carlo Simulation (MCS) are used to probabilistically analyze the failure case histories. Probabilistic analyses are needed to account for the influence of uncertainties and variabilities in soil properties and in situ test data on the reliability of the back-calculated relationship between S_{u_LIQ} and $\min(N_1)_{60}$. The probabilistic analyses provide estimates of the probability of failure P_F for cases of flow liquefaction failure in addition to the traditional factor of safety *FS* estimates. The FORM and MCS probabilistic procedures are used in conjunction with the infinite slope and Spencer's (1967) stability models.

Calculation of probability of failure requires definition of a performance function. Performance functions provide a limit surface which defines the boundary between failure and safety. Typically failure is defined as factors of safety *FS* less than 1 and safety is defined as factors of safety greater than 1. Therefore, the performance function $G(x)$ used to assess the reliability is given as:

$$G(x) = C_1 FS - 1 \tag{1}$$

where *FS* is a function of all parameters involved in the slope stability analysis and the liquefied shear strength. The C_1 term accounts for uncertainty in the performance function, and is discussed in more detail below. Failure corresponds to $G(x) \le 0$ and safety corresponds to $G(x) > 0$.

3.1 *First-Order Reliability Method*

FORM involves calculation of the reliability index β, which is a measure of the standardized distance between the "mean" point (all inputs are assigned mean values) and the failure surface. Several procedures are available to compute β most of which involve developing the first derivative of the performance function. This task can be quite cumbersome as the performance function becomes complex. Low & Tang (2004) present an ellipsoidal approach where formulation of the first derivative is not required, and correlated and non-normal parameters are easily incorporated in a spreadsheet format. Equation (2) is used to calculate the minimum β:

$$\beta = \min_{\underline{x} \in F} \sqrt{\left[\frac{x_i - m_i^N}{\sigma_i^N}\right]^T [\underline{R}]^{-1} \left[\frac{x_i - m_i^N}{\sigma_i^N}\right]} \tag{2}$$

where $\underline{x}$ is a vector representing the random variables in the slope stability calculations, F is the failure domain, $[\underline{R}]$ is the correlation matrix, and m_i^N and σ_i^N are vectors of the equivalent-normal mean and standard deviation computed from Rackwitz-Fiessler (1978) transformations. When calculating reliability indices using Equation (1), it is important to consider the deterministic FS, as only positive values of β can be obtained. If the deterministic FS is less than 1 (i.e., within the failure domain) the computed reliability index should be made negative. If the deterministic FS is greater than 1 (i.e., with the safe domain) the computed reliability index should be positive.

The probability of failure P_F is normally computed with the notional probability concept, which assumes that the probability of failure can be computed from the reliability index β according to:

$$P_F = 1 - \Phi(\beta) = \Phi(-\beta) \tag{3}$$

where $\Phi()$ is the cumulative normal distribution.

3.2 *Monte Carlo Simulation*

MCS consist of generating a large number of samples, typically in the order of 10,000 to 100,000, from probability density functions (PDFs) of the parameters involved in the slope stability calculations. MCS then calculates the performance function for each group of samples using a prescribed stability model. Several commercially available software packages, such as @RISK (Palisades 1996), can be used to perform the simulations within Microsoft Excel. The Package @RISK was used for the infinite slope stability calculations. In addition, the slope stability software SLIDE from Rocscience (2006) combines MCS with several stability models. This software was used to analyze the Spencer-type cases of flow failure. In the context of slope stability, MCS provides a distribution of the factor of safety against failure. The P_F is then computed as the area under the factor of safety probability density function less than 1, or the probability that the performance function (Equation 1) is less than zero. When the MCS and FORM models have the same setup, the resulting P_F values should be identical as shown by Low & Tang (1997).

3.3 *Uncertainties in Parameters*

For all case histories that were back-analyzed, the magnitudes of uncertainties involved in evaluating the liquefied shear strength from field data have been carefully delineated and systematically analyzed. Probability Distribution Functions (PDFs) representative of the various parameters involved in the stability analyses were developed through available data from each case history or from historical catalogs of parameter uncertainty. For those case histories where the variations in site-specific data distributions are not available, published representative values of probabilistic parameters will be used. Normal and lognormal PDFs are two widely used distributions. While most data in nature appears to follow these distributions, they both can provide unreasonable values for geotechnical problems. The normal distribution spans from negative infinity to positive infinity. When modeling parameters such as shear strength, friction angle or unit weight, negative and very high values are not reasonable. The lognormal distribution spans from zero to positive infinity. While the lognormal PDF does address the problem with negative values, very high values approaching infinity can still lead to unrealistic results and numerical problems. In addition, for most geotechnical parameters, lower bounds greater than zero are desirable. For example, the friction angle will likely never approach zero for drained sand in the field. To address the shortcomings of both normal and lognormal distributions and to avoid artificially truncating the PDFs, the Beta distribution is employed. The Beta distribution is defined by four parameters: the minimum and maximum values, the mean value, and the standard deviation. Note that truncating both the normal and lognormal distributions will also require a similar number of parameters as the Beta distribution.

4 PROBABILITIES OF FAILURE FOR CASE HISTORIES

The deterministic FS for each case is computed using the mean input values, which gives the best-fit line shown in Figure 1. The PDF of minimum SPT blow counts for each case is used to compute the distribution of liquefied shear strength from the relationship presented in Figure 1. The P_F and deterministic FS calculated for all cases are mapped using a Bayesian Mapping (BM) technique described by Juang et al. (2006). The BM procedure is based on regression analyses with the logistic function in Equation (4):

$$P_F = \frac{1}{1 + (FS / A)^B} \tag{4}$$

where A and B are mapping coefficients.

Incorporating uncertainties in the performance function with the C_1 term (Equation 4) provides a rigorous approach to computing the probability of failure. As shown by Juang et al. (2006), P_F values may be inaccurate if model uncertainties are not accounted for. To assess the model uncertainty term, parametric studies were performed by changing the mean (μ_{C1}) and standard deviation (σ_{C1}) of C_1. The values are varied until the P_F computed from the FORM or MCS match those calculated from the PDF of the reliability index. Based on the parametric studies, the model uncertainty term has a μ_{C1} of 1.0 and σ_{C1} of 0.4. Figure 2 presents the PF, which accounts for the C_1 term, as a function of FS for the liquefaction failure cases presented in Figure 1. Figure 2 shows that as the deterministic FS increases the P_F decreases, as expected. The best-fit parameters A and B are equal to 1.048 and 2.908, respectively. As can be seen, the logistic function given in Equation (4), with P_F accounting for model uncertainty through parameter C_1, provides a good representation of the mapping between P_F and FS from the back-analyzed case histories.

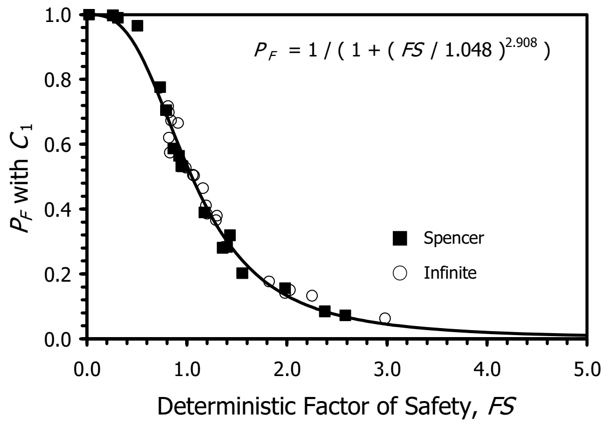

Figure 2. Mapping of P_F and FS. P_F is computed with FORM for infinite slope cases and MCS for Spencer-type cases including the C_1 term with μ of 1.0 and σ of 0.4.

5 PROBABILISTIC LIQUEFIED SHEAR STRENGTH CRITERIA

Equation (4) can be rewritten so as to plot contours of P_F on a $S_{u\text{-}LIQ}$ vs. $(N_1)_{60}$ plot:

$$\frac{S_u\left[(N_1)_{60}\right]}{FFD} = A\left(\frac{1}{P_F}-1\right)^{1/B} \tag{5}$$

where $S_u[(N_1)_{60}]$ is the liquefied shear strength from the relationship shown in Figure 1, and *FFD* is the flow failure demand, which is the shear stress acting on the post-failure geometry of failed slope. The parameter *FFD* is analogous to the parameter *CSR* (cyclic shear stress ratio) in liquefaction evaluation. The *FFD* can be estimated from the slope stability analysis. Equation (5) gives a relationship for estimating the liquefied shear strength $S_{u\text{-}LIQ}$ for a given probability of failure P_F for liquefiable cohesionless soils.

Probabilistic $S_{u\text{-}LIQ}$ versus minimum $(N_1)_{60}$ criteria are presented in Figure 3 with P_F contours corresponding to 2%, 16%, and 50%. As can be seen, the P_F=50% relationship is very close to the best fit second-degree polynomial derived in Figure 1. Figure 3 can be used in several ways to perform quick and simple probabilistic analyses of slopes and embankments containing potentially liquefiable soils: 1) with a minimum SPT blow count and *FFD*, the P_F can be estimated; 2) with a minimum SPT and a desired P_F the corresponding *FFD* can be estimated; and 3) with the minimum SPT blow count PDF, the distribution of the liquefied shear strength can be estimated for use in probabilistic slope stability calculations.

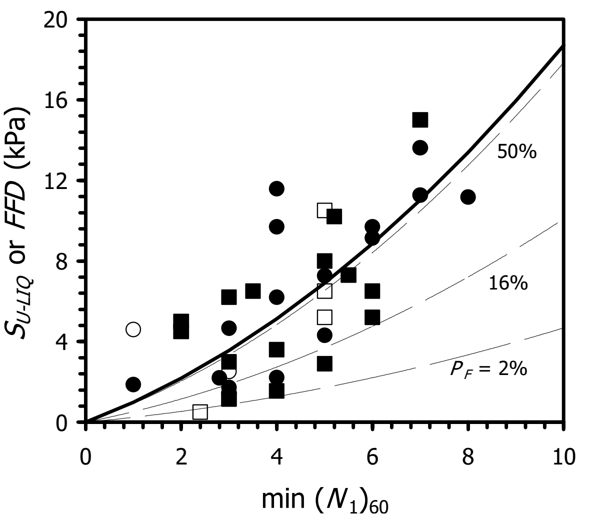

Figure 3. Contours of P_F computed with the liquefied shear strength relation shown in Fig. 1 including model uncertainty.

6 SUMMARY AND CONCLUSIONS

The paper presented probabilistic undrained residual or liquefied shear strength values of liquefiable soils as function of SPT blow count. The liquefied shear strengths were back-calculated using slope stability analysis of previous case histories of flow liquefaction failures. Probabilistic procedures, including the First-Order Reliability Method (FORM) and Monte Carlo Simulations (MCS) were used in combination with limit equilibrium methods to analyze case histories of flow failure presented in the deterministic companion paper. Depending on the post-failure geometry of the case history, either the simplified infinite slope stability analysis or the more general Spencer method of slices analysis was used in the back-analysis.

The Beta Probability Density Function was used to model the statistical distributions and uncertainties in the geotechnical parameters involved in the probabilistic analyses. For FORM, a Bayesian Mapping procedure is used where values of P_F are computed from the probability density function of the reliability indices of flow failure. The logistic mapping function is obtained by relating the deterministic factor of safety *FS* to P_F for the liquefied shear strength relationships. A parameter C_1 was introduced to account for model uncertainty in the reliability calculations.

Probabilistic $S_{u\text{-}LIQ}$ versus minimum $(N_1)_{60}$ criteria were presented for P_F contours corresponding to 2%, 16%, and 50%. It was shown that the P_F=50% relationship is very close to the best fit relations obtained from the deterministic analysis of the case histories. The probabilistic $S_{U\text{-}LIQ}$ versus minimum $(N_1)_{60}$ criteria provide a more rational procedure for estimating the post-liquefaction stability of cohesionless soils deposits by providing estimates of the probability of failure in addition to traditional values of factor of safety. The probability of failure can account for the different uncertainties in the back-calculation of the liquefied shear strength values from case histories, and the natural variability and uncertainties and properties of soil deposits.

REFERENCES

Fear, C.E. and McRoberts, E.C. 1995. Reconsideration of initiation of liquefaction in sandy soils, *Journal of Geotechnical Engineering ASCE*, Vol. 121(3), pp. 249-261.

Finn, W.D.L. 2000. Post-liquefaction flow deformations, Soil Dynamics and Liquefaction 2000, R.Y.S. Pak and J. Yamamuro (eds.), A*SCE Geotechnical Special Publication* No. 107, pp. 108-122.

Ishihara, K. 1993. Liquefaction and flow failure during earthquakes - 33rd Rankine Lecture, *Geotechnique,* Vol. 43(3), pp. 351-415.

Juang, C.H., Jiang, T. and R.D. Andrus 2002. Assessing probability-based methods for liquefaction potential evaluation, *Journal of Geotechnical and Geoenviromental Engineering ASCE*, Vol. 128(7), pp. 580-589.

Low, B.K. and Tang, W.H. 1997. Reliability analysis of reinforced embankments on soft ground, *Canadian Geotechnical Journal*, Vol. 34, pp. 672-685.

Low, B.K. and Tang, W.H. 2004. Reliability analysis using object-oriented constrained optimization, *Structural Safety*, Vol. 26, pp. 69-89.

Olson, S.M. and Stark, T.D. 2003. Liquefied strength ratio from liquefaction flow failure case histories. *Canadian Geotechnical Journal*, Vol. 39, pp. 629-647.

Palisade Corporation, 1996. @RISK: Risk analysis and simulation add-in for Microsoft Excel or Lotus 1-2-3, Palisade Corporation, Newfield, N.Y.

Poulos, S.J. 1981. The steady-state of deformation, *Journal of Geotechnical and Geoenviromental Engineering ASCE*, Vol. 107(5), pp. 553-562.

Rackwitz, R. and Fiessler, B. 1978. Structural reliability under combined random load sequences, *Computers and Structures*, Vol. 9, pp. 484–494.

Rocscience, Inc. 2006. SLIDE version 5.025: 2-D limit equilibrium analysis of slope stability, Rocscience, Inc., Toronto, Ontario.

Seed, H.B. 1987. Design problems in soil liquefaction, *Journal of Geotechnical and Geoenviromental Engineering ASCE*, Vol. 113(8), pp. 827-845.

Spencer, E. 1967. A method of analysis of the stability of embankments assuming parallel interslice forces, *Géotechnique*, Vol. 17(1), pp. 11-26.

Stark, T.D., Kramer, S.L. and Youd, T.L. 1997. Post-liquefaction shear strength of granular soils, *Proceedings NSF Workshop Post-liquefaction Shear Strength of Granular Soils*, National Science Foundation Grant CMS-95-31678, unpublished.

Wride (Fear), C.E., McRoberts, E.C. and Robertson, P.K. 1999. Reconsideration of case histories for estimating undrained shear strength in sandy soils, *Canadian Geotechnical Journal*, Vol. 36, pp: 907-933.