ECAI 2008
M. Ghallab et al. (Eds.)
IOS Press, 2008

doi:10.3233/978-1-58603-891-5-89

Conservativity in Structured Ontologies[1]

Oliver Kutz[2] and **Till Mossakowski**[3]

Abstract. Using category theoretic notions, in particular diagrams and their colimits, we provide a common semantic backbone for various notions of modularity in structured ontologies, and outline a general approach for representing (heterogeneous) combinations of ontologies through interfaces of various kinds, based on the theory of institutions. This covers theory interpretations, (definitional) language extensions, symbol identifications, and conservative extensions. In particular, we study the problem of inheriting conservativity between sub-theories in a diagram to its colimit ontology, and apply this to the problem of localisation of reasoning in 'modular ontology languages' such as DDLs or $\mathcal{E}$-connections.

1 Introduction

In this paper, we propose to use the category theoretic notions of diagram and colimit in order to provide a common semantic backbone for various notions of modularity in ontologies.[4]

At least three commonly used notions of 'module' in ontologies can be distinguished, depending on the kind of relationship between the 'module' and its supertheory (or superontology): (1) a module can be considered a 'logically independent' part within its superontology—this leads to the definition of module as a part of a larger ontology which is a conservative extensions of it; (2) a module can be a part of a larger 'integrated ontology', where the kind of integration determines the relation between the modules—this is the approach followed by modular ontology languages (e.g. DDLs, $\mathcal{E}$-connections etc.); (3) a 'part' of a larger theory can be considered a module for reasons of elegance, re-use, tradition, etc.—in this case, the relation between a module and its supertheory might be a language extension, theory extension/interpretation, etc.

The main contributions of the present paper are the following: (i) building on the theory of institutions, diagrams, and colimits, we show how these different notions of module can be considered simultaneously using the notion of a *module diagram*; (ii) we show how conservativity properties can be traced and inherited to the colimit of a diagram; (iii) we show how this applies to the composition problem in modular ontology languages such as DDLs and $\mathcal{E}$-connections.

Section 2 introduces institutions, Section 3 the diagrammatic view of modules, and Section 4 studies the problem of conservativity in diagrams. Finally, we sketch heterogeneous diagrams and apply this to modular ontology languages in Section 5.[5]

2 Institutions

The study of modularity principles can be carried out to a quite large extent independently of the details of the underlying logical system that is used. The notion of **institutions** was introduced by Goguen and Burstall in the late 1970s exactly for this purpose (see [14]). They capture in a very abstract and flexible way the notion of a logical system by describing how, in any logical system, signatures, models, sentences (axioms) and satisfaction (of sentences in models) are related. The importance of the notion of institutions lies in the fact that a surprisingly large body of logical notions and results can be developed in a way that is completely independent of the specific nature of the underlying institution.[6]

An **institution** $I = (\mathbf{Sign}, \mathbf{Sen}, \mathbf{Mod}, \models)$ consists of (i) a category $\mathbf{Sign}$ of *signatures*; (ii) a functor $\mathbf{Sen}\colon \mathbf{Sign} \longrightarrow \mathbf{Set}$ giving, for each signature Σ, the set of *sentences* $\mathbf{Sen}(\Sigma)$, and for each signature morphism $\sigma\colon \Sigma \longrightarrow \Sigma'$, the *sentence translation map* $\mathbf{Sen}(\sigma)\colon \mathbf{Sen}(\Sigma) \longrightarrow \mathbf{Sen}(\Sigma')$, where $\mathbf{Sen}(\sigma)(\varphi)$ is abbreviated $\sigma(\varphi)$; (iii) a functor $\mathbf{Mod}\colon \mathbf{Sign}^{op} \longrightarrow \mathcal{CAT}$ giving, for each signature Σ, the category of *models* $\mathbf{Mod}(\Sigma)$, and for each signature morphism $\sigma\colon \Sigma \longrightarrow \Sigma'$, the *reduct functor* $\mathbf{Mod}(\sigma)\colon \mathbf{Mod}(\Sigma') \longrightarrow \mathbf{Mod}(\Sigma)$, where $\mathbf{Mod}(\sigma)(M')$ is abbreviated $M'|_\sigma$; (iv) a satisfaction relation $\models_\Sigma \subseteq |\mathbf{Mod}(\Sigma)| \times \mathbf{Sen}(\Sigma)$ for each $\Sigma \in |\mathbf{Sign}|$, such that for each $\sigma\colon \Sigma \longrightarrow \Sigma'$ in $\mathbf{Sign}$ the following *satisfaction condition* holds:

$$(\star) \qquad M' \models_{\Sigma'} \sigma(\varphi) \text{ iff } M'|_\sigma \models_\Sigma \varphi$$

for each $M' \in |\mathbf{Mod}(\Sigma')|$ and $\varphi \in \mathbf{Sen}(\Sigma)$, expressing that truth is invariant under change of notation and enlargement of context.

The only condition governing the behaviour of institutions is thus the *satisfaction condition* $(\star)$.[7]

A **theory** in an institution is a pair $T = (\Sigma, \Gamma)$ consisting of a signature $\mathsf{Sig}(T) = \Sigma$ and a set of Σ-sentences $\mathsf{Ax}(T) = \Gamma$, the axioms of the theory. The models of a theory T are those $\mathsf{Sig}(T)$-models that satisfy all axioms in $\mathsf{Ax}(T)$. Logical consequence is defined as usual: $T \models \varphi$ if all T-models satisfy φ. **Theory morphisms**, also called **interpretations of theories**, are signature morphisms that map axioms to logical consequences.

Examples of institutions include first- and higher-order classical logic, description logics, and various (quantified) modal logics [19].

3 Modules as Diagrams

Several approaches to modularity in ontologies have been discussed in recent years, including the introduction of various so-called 'modular ontology languages'. The module system of the Web Ontology

[1] Work on this paper has been supported by the Vigoni program of the DAAD and by the DFG-funded collaborative research centre 'Spatial Cognition'.
[2] SFB/TR 8 Spatial Cognition, University of Bremen, Germany. okutz@informatik.uni-bremen.de
[3] DFKI GmbH, Bremen, Germany, till.mossakowski@dfki.de
[4] This paper extends the results of [18].
[5] Proofs for the results of this paper can be found in [19].
[6] For an extensive treatment of the model theory in this setting, see [10].
[7] Note, however, that non-monotonic formalisms can only indirectly be covered this way, but compare, e.g., [16].

Language OWL itself is as simple as inadequate [9]: it allows for importing other ontologies, including cyclic imports. The language CASL, originally designed as a first-order algebraic specification language, is used for ontologies in [21]. Beyond imports, it allows for renaming, hiding and parameterisation. Other languages envisaging more involved integration and modularisation mechanisms than plain imports include DDLs [6], $\mathcal{E}$-connections [17], and P-DLs [4].

We will use the formalism of colimits of diagrams as a common semantic backbone for these languages.[8] The intuition behind colimits is explained as follows:

> "Given a species of structure, say widgets, then the result of interconnecting a system of widgets to form a super-widget corresponds to taking the *colimit* of the diagram of widgets in which the morphisms show how they are interconnected." [13]

The notion of **diagram** is formalised in category theory. Diagrams map an index category (via a functor) to a given category of interest. They can be thought of as graphs in the category. A **cocone** over a diagram is a kind of "tent": it consists of a tip, together with a morphism from each object involved in the diagram into the tip, such that the triangles arising from the morphisms in the diagram commute. A **colimit** is a universal, or minimal cocone. For details, see [1].

In the sequel, we will assume that the signature category has all finite colimits, which is a rather mild assumption; in particular, it is true for all the examples of institutions mentioned above. Moreover, we will rely on the fact that colimits of theories exist in this case as well; the colimit theory is defined as the union of all component theories in the diagram, translated along the signature morphisms of the colimiting cocone.

Definition 1 *A* ***module diagram*** *of ontologies is a diagram of theories such that the nodes are subdivided into* ontology nodes *and* interface nodes*.*

Composition of module diagrams is simply their union.

Example 1 *Consider the union of the diagrams*

$$\begin{tikzcd}[column sep=small]
T_1 & & T_2 & T_2 & & T_3 \\
& \Sigma_1 \arrow[ul] \arrow[ur] & & & \Sigma_2 \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

where the Σ_i are interfaces and the T_i are ontologies. Think of e.g. T_{12} as being an ontology that imports T_1 and T_2, where Σ_1 contains all the symbols shared between T_1 and T_2. Then T_{12} (and T_{23}) can be obtained as pushouts, and so can the overall union T_{123} (which typically is then further extended with new concepts etc.). A "c" means "conservative"; this will be explained in Sect. 4.

$$\begin{tikzcd}
& & T_{123} & & \\
& T_{12} \arrow[ur, dashed, "c"] & & T_{23} \arrow[ul, dashed, "c"'] & \\
T_1 \arrow[ur, dashed, "c"] & & T_2 \arrow[ul, dashed, "c"'] \arrow[ur, dashed, "c"] & & T_3 \arrow[ul, dashed, "c"'] \\
& \Sigma_1 \arrow[ul] \arrow[ur] & & \Sigma_2 \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Notice that Example 1 is closely related to the composition (or combination) of ontology alignments, as introduced in [26], and further studied in [20]. In general, it is clear that theories with an import structure are just tree-shaped diagrams, while both shared parts and cyclic imports lead to arbitrary graph-shaped diagrams. The translation of CASL (without hiding) to so-called development graphs detailed in [7] naturally leads to diagrams as well. Finally, the diagrams corresponding to modular languages like DDLs and $\mathcal{E}$-connections will be studied in Sect. 5. Thus, diagrams can be seen as a uniform mathematical formalism where properties of all of these module concepts can be studied. An important such property is conservativity.

4 Conservative Diagrams and Composition

Conservative diagrams are important because they imply that the combined ontology does not add new facts to the individual ontologies. Indeed, the notion of an ontology module of an ontology T has been defined as any "subontology T' such that T is a conservative extension of T'" [12]—this perfectly matches our notion of conservative diagram below.

Definition 2 *A theory morphism $\sigma\colon T_1 \longrightarrow T_2$ is* ***proof-theoretically conservative****, if T_2 does not entail anything new w.r.t. T_1, formally, $T_2 \models \sigma(\varphi)$ implies $T_1 \models \varphi$. Moreover, $\sigma\colon T_1 \longrightarrow T_2$ is* ***model-theoretically conservative****, if any T_1-model M_1 has a σ-expansion to T_2, i.e. a T_2-model M_2 with $M_2|_\sigma = M_1$.*

It is easy to show that conservative theory morphisms compose. Moreover, model-theoretic implies proof-theoretic conservativity. However, the converse is not true in general, compare [22] for an example.

Definition 3 *A* ***(proof-theoretic, model-theoretic) conservative module diagram*** *of ontologies is a diagram of theories such that the theory morphism of any ontology node into the colimit of the diagram is (proof-theoretically resp. model-theoretically) conservative.*

By conservativity, the definition immediately yields:

Proposition 1 *The colimit ontology of a proof-theoretic (model-theoretic) conservative module diagram is consistent (satisfiable)*[9] *if any of the component ontologies is.*

Thus, in particular, in a conservative module diagram, an ontology node O_i can only be consistent (satisfiable) if all other ontology nodes O_j, $j \neq i$, are consistent (satisfiable) as well.

The main question is how to ensure these conservativity properties in the united diagram. To study this, we introduce some notions from model theory, namely various notions of **interpolation** (for proof-theoretic conservativity) and **amalgamation** (for model-theoretic conservativity).

Craig interpolation plays a crucial role in connection with proof systems in structured theories. The most common formulation, i.e. **Craig** (or Arrow) interpolation, however, relies on a connective $\rightarrow$ being present in the institution. A slightly more general formulation, often called **turnstile** interpolation is as follows: if $\varphi \models \psi$, then there exists some χ that only uses symbols occurring in both φ and ψ, with $\varphi \models \chi$ and $\chi \models \psi$. This, of course, follows from Craig interpolation in the presence of a deduction theorem.

For the general study of module systems, we need to generalise such definitions in at least two important ways. The first concerns the

[8] However, note that hiding is not covered by this approach.

[9] Contrary to the terminology used in DL, we distinguish here proof-theoretic (syntactic) consistency of a theory T (which means $T \not\models \varphi$ for some sentence φ) from model-theoretic (semantic) satisfiability (which means $M \models T$ for some model M).

rather implicit use of signatures in the standard definitions. Making signatures explicit means to assume that φ lives in a signature Σ_1, ψ lives in a signature Σ_2, the entailment $\varphi \models \psi$ lives in $\Sigma_1 \cup \Sigma_2$, and the interpolant in $\Sigma_1 \cap \Sigma_2$. Since we do not want to go into the technicalities for equipping an institution with unions and intersections (see [11] for details), we replace $\Sigma_1 \cap \Sigma_2$ with a signature Σ, and $\Sigma_1 \cup \Sigma_2$ with Σ' such that Σ' is obtained as a pushout from the other signatures via suitable signature morphisms (cf. the diagram below). Secondly, we move from single sentences to *sets* of sentences. This is useful since we want to support DLs and TBox reasoning, and DLs like (sub-Boolean) $\mathcal{EL}$ do not allow to rewrite 'conjunctions of subsumptions', i.e., we cannot collapse a TBox into a single sentence. (In case of compact logics, the use of sets is equivalent to the use of finite sets.)

This leads to the following definition. In the sequel, fix an arbitrary institution $I = (\mathbf{Sign}, \mathbf{Sen}, \mathbf{Mod}, \models)$:

Definition 4 *The institution I has the* ***Craig-Robinson interpolation property (CRI*** *for short), if for any pushout*

$$\begin{tikzcd} & \Sigma' & \\ \Sigma_1 \arrow[ur, dashed, "\theta_1"] & & \Sigma_2 \arrow[ul, dashed, "\theta_2"'] \\ & \Sigma \arrow[ul, "\sigma_1"] \arrow[ur, "\sigma_2"'] & \end{tikzcd}$$

any set Γ_1 of Σ_1-sentences and any sets Γ_2, Δ_2 of Σ_2-sentences with

$$\theta_1(\Gamma_1) \cup \theta_2(\Delta_2) \models \theta_2(\Gamma_2),$$

there exists a set of Σ-sentences Γ (called the interpolant*) such that*

$$\Gamma_1 \models \sigma_1(\Gamma) \textit{ and } \Delta_2 \cup \sigma_2(\Gamma) \models \Gamma_2.$$

CRI, in general, is strictly stronger than Craig interpolation. However, for almost all logics typically used in knowledge representation, they are indeed equivalent. We give a criterion that applies to institutions generally, taken from [10]:

Proposition 2 *A compact institution with implication has CRI iff it has Craig interpolation.*

Here, an institution I **has implication** if for any two Σ-sentences φ, ψ, there exists a Σ-sentence χ such that, for any Σ-model M,

$$M \models \chi \text{ iff } (M \models \varphi \text{ implies } M \models \psi)$$

Moreover, I is **compact** if $T \models \varphi$ implies $T' \models \varphi$ for a finite subtheory T' of T. Since for modal logics, the deduction theorem (for the global consequence relation $\models$) generally fails, these logics do not have implication in the above sense, and we cannot apply Prop. 2. However, various more specialised criteria can be given, see [19]. Some results are summarised in Fig. 1.

The amalgamation property (called 'exactness' in [11]) is a major technical assumption in the study of specification semantics, see [23].

Definition 5 *An institution I is* ***(weakly) exact*** *if, for any diagram of signatures, any* compatible *family of models (i.e. compatible with the reducts induced by the involved signature morphisms) can can be* amalgamated *to a unique (or weakly amalgamated to a not necessarily unique) model of the colimit. For pushouts, this amounts to the following (we use notation as in Def. 4): any pair $(M_1, M_2) \in \mathbf{Mod}(\Sigma_1) \times \mathbf{Mod}(\Sigma_2)$ that is compatible (in the sense that M_1 and M_2 reduce to the same Σ-model) can be amalgamated to a (unique) Σ'-model M (i.e., there exists a (unique) $M \in \mathbf{Mod}(\Sigma')$ that reduces to M_1 and M_2, respectively).*

Institution	weakly exact	exact	CRI
$\mathcal{EL}$	+	-	+
$\mathcal{ALC}^{\mathsf{ms}}$	+	+	+
$\mathcal{ALC}$	+	-	+
$\mathcal{ALCO}$	+	-	-
$\mathcal{ALCQO}$	+	-	-
$\mathcal{SHOIN}$	+	-	-
$\mathbf{FOL}^{\mathsf{ms}}$	+	+	+
$\mathbf{QS5}$	+	-	-

Figure 1. (Weak) exactness and Craig-Robinson interpolation

Weak exactness for these institutions follows with standard methods, see [10]. The same holds for exactness for the many-sorted variants. Exactness, however, obviously fails for the single-sorted logics as well as for $\mathbf{QS5}$, because in these logics, the implicit universe resp. the implicit set of worlds leads to the phenomenon that the empty signature has many different models. The following propositions are folklore in institutional model theory, see [10].

Theorem 1 *1. In an institution with CRI proof-theoretic conservativity is preserved along pushouts.*

2. In an institution that is weakly exact, model-theoretic conservativity is preserved along pushouts.

We now give necessary conditions for the preservation of conservativity when taking the colimit of the union of conservative diagrams.

Firstly, a diagram is **thin**, or a **preorder**, if its index category is thin (i.e., there is at most one arrow between two given objects). Consider the following non-thin union diagram (assuming that the two arrows in the union are inherited from two different ontologies), where $\{P \sqsubseteq \top\} \subseteq T_1$ and $\{C_1 \equiv \neg C_2\} \subseteq T_2$:

$$\begin{tikzcd} T_1 \arrow[r, shift left, "P \mapsto C_1"] \arrow[r, shift right, "P \mapsto C_2"'] & T_2 \arrow[r, dotted] & T_3 \supseteq C \equiv \neg C \end{tikzcd}$$

Although the individual ontologies are conservative, the union is not because in the colimit C_1 and C_2 are identified.

Next, a preorder is **finitely bounded inf-complete** if any two elements with a common lower bound have an infimum. Consider the following, not finitely bounded inf-complete union diagram (assume that it is obtained as the union of its upper and its lower half):

$$\begin{tikzcd} & P \sqsubseteq Q \arrow[rr, dotted] & & P \equiv Q \\ P \sqsubseteq \top \arrow[ur, "c"] \arrow[dr, "c"'] & & Q \sqsubseteq \top \arrow[ul, "c"'] \arrow[dl, "c"] & \\ & Q \sqsubseteq P \arrow[uurr, dotted] & & \end{tikzcd}$$

Again, the individual ontologies are conservative, but the colimit of the union is not. Hence, call a diagram **tame** if it does not show these sources of inconsistency/non-conservativity, i.e. if it is thin and finitely bounded inf-complete.

Theorem 2 *1. Assume institution I has an initial signature*[10] *and has CRI (is weakly exact). If the involved ontologies are consistent (satisfiable), then composition of module diagrams via union preserves proof-theoretic (model-theoretic) conservativity if the diagram resulting from the union of the individual diagrams and their colimits is tame.*

2. If the union is a disjoint union, the tameness assumption can be dropped.

[10] Usually, the empty signature is initial.

Note that consistency of the involved ontologies can be replaced with connectedness of the united diagram.
The above examples and Example 2 below show that the conditions from the theorem are essentially optimal. See Example 1 for a conservative union of conservative diagrams.

5 Heterogeneity and Modular Languages

As [24] argue convincingly, relating or integrating ontologies may happen across different institutions as ontologies are written in many different formalisms, like relation schemata, description logics, first-order logic, and modal logics.

Heterogeneous specification is based on some graph of logics and logic translations, formalised as institutions and so-called institution comorphisms, see [15]. The latter are again governed by the satisfaction condition, this time expressing that truth is invariant also under change of notation across different logical formalisms:

$$M' \models^J_{\Phi(\Sigma)} \alpha_\Sigma(\varphi) \Leftrightarrow \beta_\Sigma(M') \models^I_\Sigma \varphi.$$

Here, $\Phi(\Sigma)$ is the translation of signature Σ from institution I to institution J, $\alpha_\Sigma(\varphi)$ is the translation of the Σ-sentence φ to a $\Phi(\Sigma)$-sentence, and $\beta_\Sigma(M')$ is the translation (or perhaps: reduction) of the $\Phi(\Sigma)$-model M' to a Σ-model. The definitions and results of the previous sections also apply to the heterogeneous case. However, special care is needed in obtaining CRI or (weak) exactness [10].

Heterogeneous knowledge representation was also a major motivation for the definition of modular languages, $\mathcal{E}$-connections in particular [17]. We here show how the integration of ontologies via 'modular languages' can be re-formulated in module diagrams. In the following, we will assume basic acquaintance with the syntax and semantics of both, DDLs and $\mathcal{E}$-connections, which we reformulate as many-sorted theories. Details have to remain sketchy for lack of space.

It should be clear that DDLs or $\mathcal{E}$-connections can essentially be considered as many-sorted heterogeneous theories: component ontologies can be formulated in different logics, but have to be built from many-sorted vocabulary, and link relations are interpreted as relations connecting the sorts of the component logics (compare [3] who note that this is an instance of a more general co-comma construction). To be more precise, assume a DDL $\mathfrak{D} = (\mathcal{S}_1, \mathcal{S}_2)$ is given. Knowledge bases for $\mathfrak{D}$ can contain **bridge rules** of the form:

$$C_i \xrightarrow{\sqsubseteq} C_j \quad \text{(into rule)} \qquad C_i \xrightarrow{\sqsupseteq} C_j \quad \text{(onto rule)}$$

where C_i and C_j are concepts from $\mathcal{S}_i$ and $\mathcal{S}_j$ ($i \neq j$), respectively (we consider here only DDL in its most basic form without individual correspondences etc.).

An interpretation $\mathfrak{I}$ for a DDL knowledge base is a pair $(\{\mathcal{I}_i\}_{i \leq n}, \mathcal{R})$, where each $\mathcal{I}_i$ is a model for the corresponding $\mathcal{S}_i$, and $\mathcal{R}$ is a function associating with every pair (i, j), $i \neq j$, a binary relation $r_{ij} \subseteq W_i \times W_j$ between the domains W_i and W_j of $\mathcal{I}_i$ and $\mathcal{I}_j$, respectively.

In the many-sorted re-formulation of DDLs, the relation r_{ij} is now interpreted as a relation between the $\top$-sort of $\mathcal{S}_1$ and the $\top$-sort of $\mathcal{S}_2$. Bridge rules are expressed as existential restrictions of the form

$$(\sharp) \quad \exists r_{ij}.C_i \sqsubseteq C_j \qquad \text{and} \qquad \exists r_{ij}.C_i \sqsupseteq C_j$$

The fact that bridge rules are atomic statements in a DDL knowledge base now translates to a restriction on the grammar governing the usage of the link relation r_{ij} in the multi-sorted formalism (see [5] for a discussion of related issues). In fact, the main difference between DDLs and various $\mathcal{E}$-connections now lies in the expressivity of this 'link language' $\mathcal{L}$ connecting the different sorts of the ontologies. In basic DDL as defined above, the only expressions allowed are those given in ($\sharp$), so the link language of basic DDL is a certain, very weak sub-Boolean fragment of many sorted $\mathcal{ALC}$, namely the one given through ($\sharp$). In $\mathcal{E}$-connections, expressions of the form $\exists r_{ij}.C_i$ are again concepts of $\mathcal{S}_j$, to which Booleans (or other operators) of $\mathcal{S}_j$ as well as restrictions using relations r_{ji} can be applied. Thus, the basic link language of $\mathcal{E}$-connections is sorted $\mathcal{ALCI}^{\mathsf{ms}}$(relative to the now richer languages of $\mathcal{S}_i$).[11]

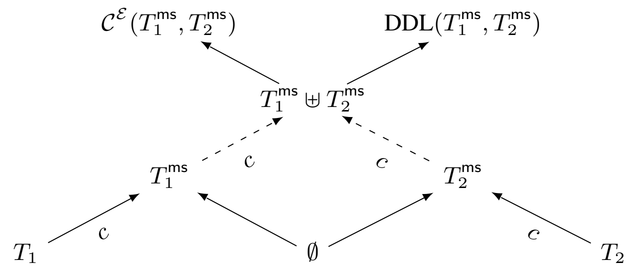

Figure 2. $\mathcal{E}$-connections and DDLs many-sorted

Such many-sorted theories can easily be represented in a diagram as shown in Figure 2. Here, we first (conservatively) obtain a disjoint union $T_1^{\mathsf{ms}} \uplus T_2^{\mathsf{ms}}$ as a pushout, where the component ontologies have been turned into sorted variants (using an institution comorphism from the single-sorted to the many-sorted logic), and the empty interface guarantees that no symbols are shared at this point. An $\mathcal{E}$-connection KB in language $\mathcal{C}^{\mathcal{E}}(T_1^{\mathsf{ms}}, T_2^{\mathsf{ms}})$ or a DDL KB in language $\mathrm{DDL}(T_1^{\mathsf{ms}}, T_2^{\mathsf{ms}})$ is then obtained as a (typically not conservative) theory extension.

When connecting ontologies via bridges, or interfaces, this typically is not conservative everywhere, but only for some of the involved ontologies. We give a criterion for a single ontology to be conservative in the combination. While the theorem can be applied to arbitrary interface nodes, when applied to $\mathcal{E}$-connections or DDLs, we assume that bridge nodes contain DDL bridge rules or $\mathcal{E}$-connection assertions.

Theorem 3 *Assume that we work in an institution that has CRI (is weakly exact). Let ontologies $T_1, \ldots, T_n$ be connected via bridges B_{ij}, $i < j$. If T_i is proof-theoretically (model-theoretically) conservative in B_{ij} for $j > i$, then T_1 is proof-theoretically (model-theoretically) conservative in the resulting colimit ontology T.*

The diagram in Fig. 3 illustrates Theorem 3 for the case $n = 3$.

As concerns the applicability of the theorem, we have given an overview of logics being (weakly) exact or having CRI in Fig. 1. Of course, the conservativity assumptions have to be shown additionally.

We next give an example of the failure of the claim of the theorem in case we work in a logic that lacks Craig-Robinson interpolation.

Example 2 *The presence of nominals in description or modal logics generally destroys (standardly formulated) Craig interpolation [2]. Here is a counterexample for the logic $\mathcal{ALCO}$. Let*

$$\begin{array}{rcl} \Gamma & := & \{\top \sqsubseteq \exists S.C \sqcap \exists S.\neg C\} \text{ and} \\ \Delta & := & \{\forall S.(D \sqcup i) \sqsubseteq \exists S.D\} \end{array}$$

[11] But can be weakened to $\mathcal{ALC}^{\mathsf{ms}}$ or the link language of DDLs, or strengthened to more expressive many-sorted DLs such as $\mathcal{ALCQI}^{\mathsf{ms}}$.

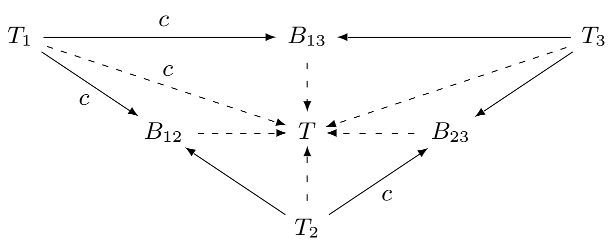

Figure 3. Colimit integration along bridges for $n = 3$

where i is a nominal. Clearly, $\Gamma \models \Delta$, for in every model $M \models \Gamma$, every point has at least two S-successors. But i can only be true in at most one of those successors, which entails $M \models \Delta$. Now, (using bisimulations) it can be shown that in $\mathcal{ALCO}$ there is no Δ' built from shared concept names alone (there are none) such that $\Gamma \models \Delta'$ and $\Delta' \models \Delta$.

Assume now ontologies T_1, T_2, T_3 are formulated in the DL $\mathcal{ALCO}$ with signatures $\mathsf{Sig}(T_1) \subseteq \{S, B, D, i\}$, $\mathsf{Sig}(T_2) \subseteq \{C_1, C_2\}$, and $\mathsf{Sig}(T_3) \subseteq \{B_1, B_2\}$. Also, assume $\{\exists S.D\} \subseteq T_1$.

Consider now the situation depicted in Fig. 3 with

$$\begin{aligned} B_{12} &\supseteq \{\top \sqsubseteq \exists S.\exists R_1.C_1, \top \sqsubseteq \exists S.\exists R_1.\neg C_2\}, \\ B_{13} &\supseteq \{B_1 \equiv \exists R_3^{-1}.B, B_2 \equiv \exists R_3^{-1}.B\}, \\ B_{23} &\supseteq \{C_1 \equiv \exists R_2.B_1, C_2 \equiv \exists R_2.B_2\}. \end{aligned}$$

Here, the roles R_1, R_2, R_3 can be seen as link relations, and since we apply existential restrictions $\exists S$ to $\exists R_2.C_1$ etc., the example can be understood as a composition of (binary) $\mathcal{E}$-connections.

The reader can check that T_i is conservative in B_{ij} for $j > i$. However, in the colimit (union) of this diagram, $\forall S.D \sqcup i \sqsubseteq \exists S.D$ follows, while this does not follow in T_1, and thus T_1 is not conservative in the colimit ontology.

Thus, if the assumptions of the theorem are satisfied, reasoning over the signature of T_1 can be performed *within* T_1, i.e. without considering the overall integration T. This, however, can not be guaranteed for logics lacking CRI. In the light of this example, it should now come as no surprise that attempts to localise reasoning in DDLs in a peer-to-peer like fashion whilst remaining sound and complete have been restricted to logics lacking nominals [25].

6 Discussion and Outlook

Diagrams and their colimits offer the right level of abstraction to study conservativity issues in different languages for modular ontologies. We have singled out conditions that allow for lifting conservativity properties from individual diagrams to their combinations.

An interesting point is the question whether proof-theoretic or model-theoretic conservativity should be used. The model-theoretic notion ensures 'modularity' in more logics than the proof-theoretic one since the lifting theorem for the former only depends on mild amalgamation properties. By contrast, for the latter one needs Craig-Robinson interpolation which fails, e.g., for some description logics with nominals, and also for $\mathbf{QS5}$—but these logics are used in practice for ontology design.

Moreover, when relating ontologies across different institutions, the model-theoretic notion is more feasible. Finally, it has the advantage of being independent of the particular language, which implies avoidance of examples like the one presented in [22], where a given ontology extension is proof-theoretically conservative in $\mathcal{EL}$ but not in $\mathcal{ALC}$. Of course, model-theoretic conservativity generally is harder to decide, but it can be ensured by syntactic criteria, and the work related to this is promising [8].

REFERENCES

[1] J. Adámek, H. Herrlich, and G. Strecker, *Abstract and Concrete Categories*, Wiley, New York, 1990.
[2] C. Areces and M. Marx, 'Failure of interpolation in combined modal logics', *Notre Dame Journal of Formal Logic*, **39**(2), 253–273, (1998).
[3] F. Baader and S. Ghilardi, 'Connecting Many-Sorted Theories', *The Journal of Symbolic Logic*, **72**(2), 535–583, (2007).
[4] J. Bao, D. Caragea, and V. Honavar, 'On the Semantics of Linking and Importing in Modular Ontologies', in *Proc. of ISWC*. Springer, (2006).
[5] A. Borgida, 'On Importing Knowledge from DL Ontologies: some Intuitions and Problems', in *Proc. of DL*, (2007).
[6] A. Borgida and L. Serafini, 'Distributed Description Logics: Assimilating Information from Peer Sources', *Journal of Data Semantics*, **1**, 153–184, (2003).
[7] CoFI (The Common Framework Initiative), CASL *Reference Manual*, LNCS Vol. 2960 (IFIP Series), Springer, 2004. Freely available at `http://www.cofi.info`.
[8] B. Cuenca Grau, I. Horrocks, Y. Kazakov, and U. Sattler, 'Modular Reuse of Ontologies: Theory and Practice', *Journal of Artificial Intelligence Research (JAIR)*, **31**, (2008). to appear.
[9] B. Cuenca Grau, B. Parsia, and E. Sirin, 'Combining OWL Ontologies Using $\mathcal{E}$-Connections', *Journal of Web Semantics*, **4**(1), 40–59, (2006).
[10] R. Diaconescu, *Institution-independent Model Theory*, Studies in Universal Logic, Birkhäuser, 2008.
[11] R. Diaconescu, J. Goguen, and P. Stefaneas, 'Logical Support for Modularisation', in *2nd Workshop on Logical Environments*, 83–130, CUP, New York, (1993).
[12] S. Ghilardi, C. Lutz, and F. Wolter, 'Did I Damage My Ontology? A Case for Conservative Extensions in Description Logics', in *Proc. of KR-06*, pp. 187–197, (2006).
[13] J. A. Goguen, 'A categorical manifesto', *Mathematical Structures in Computer Science*, **1**, 49–67, (1991).
[14] J. A. Goguen and R. M. Burstall, 'Institutions: Abstract Model Theory for Specification and Programming', *J. of the ACM*, **39**, 95–146, (1992).
[15] J. A. Goguen and G. Roşu, 'Institution morphisms', *Formal aspects of computing*, **13**, 274–307, (2002).
[16] S. Guerra, 'Composition of Default Specifications', *Journal of Logic and Computation*, **11**(4), 559–578, (2001).
[17] O. Kutz, C. Lutz, F. Wolter, and M. Zakharyaschev, '$\mathcal{E}$-Connections of Abstract Description Systems', *Artificial Intelligence*, **156**(1), 1–73, (2004).
[18] O. Kutz and T. Mossakowski, 'Modules in Transition: Conservativity, Composition, and Colimits', in *2nd Int. Workshop on Modular Ontologies (WoMO-07)*, (2007).
[19] O. Kutz and T. Mossakowski, 'Conservativity in Structured Ontologies', Technical report, University of Bremen, `www.informatik.uni-bremen.de/~okutz/OntoStruc-TR.pdf`, (2008).
[20] O. Kutz, T. Mossakowski, and M. Codescu, 'Shapes of Alignments: Construction, Combination, and Computation', in *Int. Workshop on Ontologies: Reasoning and Modularity (WORM-08)*, (2008).
[21] K. Lüttich, C. Masolo, and S. Borgo, 'Development of Modular Ontologies in CASL', in *1st Workshop on Modular Ontologies 2006*, eds., P. Haase, V. Honavar, O. Kutz, Y. Sure, and A. Tamilin, volume 232 of *CEUR Workshop Proceedings*. CEUR-WS.org, (2006).
[22] C. Lutz and F. Wolter, 'Conservative Extensions in the Lightweight Description Logic $\mathcal{EL}$', in *Proc. of CADE-07*, pp. 84–99. Springer, (2007).
[23] D. Sannella and A. Tarlecki, 'Specifications in an arbitrary institution', *Information and Computation*, **76**, 165–210, (1988).
[24] M. Schorlemmer and Y. Kalfoglou, 'Institutionalising Ontology-Based Semantic Integration', *Journal of Applied Ontology.*, (2008). To appear.
[25] A. Tamilin, *Distributed Ontological Reasoning: Theory, Algorithms, and Applications*, Ph.D. dissertation, University of Trento, 2007.
[26] A. Zimmermann, M. Krötzsch, J. Euzenat, and P. Hitzler, 'Formalizing Ontology Alignment and its Operations with Category Theory', in *Proc. of FOIS*, pp. 277–288, (2006).