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Chapter 12

Superluminal Waves and Objects: Theory and Experiments. A Panoramic Introduction[1]

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1. Introduction

The question of superluminal ($V^2 > c^2$) objects or waves, has a long history, starting perhaps in 50 BC with Lucretius' *De Rerum Natura*.[2] Still in pre-relativistic times, one meets various related works, from those by J.J. Thomson to papers by A. Sommerfeld.

With special relativity, however, since 1905 the conviction spread that the speed c of light in vacuum was the *upper* limit of any possible speed.

For instance, in 1917 R.C. Tolman believed that he had shown by his "paradox" that the existence of particles endowed with speeds larger than c would have allowed information to be sent into the past. Such a conviction blocked for more than half a century – aside from an isolated paper (1922) by the Italian mathematician G. Somigliana – any research about superluminal speeds. Our problem started to be tackled again in the 1950s and 1960s, in particular after the papers by E.C. George Sudarshan *et al.* [7], and, later on, [38] by E. Recami, R. Mignani *et al.* (who, in their works at the beginning of the 1970s, brought the expressions subluminal and superluminal into popular use), as well as by H.C. Corben and others (to confine ourselves to the *theoretical* researches). The first experiments looking for faster-than-light objects were performed by T. Alväger *et al.* [38].

Superluminal objects were called *tachyons*, T, by G. Feinberg, from the Greek word $\tau\alpha\chi\acute{\upsilon}\varsigma$, quick (and this induced the present author in 1970 to coin the term "bradyon", for ordinary subluminal ($v^2 < c^2$) objects, from the Greek word $\beta\rho\alpha\delta\acute{\upsilon}\varsigma$, slow). Finally, objects travelling exactly at the speed of light are called "luxons".

In recent years, terms as "tachyon" and "superluminal" fell unhappily into the (cunning, rather than crazy) hands of pranotherapists and mere cheats, who started squeezing money out of simple-minded people; for instance by selling plasters (!) that should cure

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[2] Cf. e.g. book 4, line 201: "Quone vides *citius* debere et longius ire/Multiplexque loci spatium transcurrere eodem/Tempore *quo Solis* pervolgant *lumina* coelum?"

various illnesses by "emitting tachyons"... We are dealing with tachyons here, however, since at least four different experimental sectors of physics seem to indicate the actual existence of superluminal motions (thus confirming some long-standing theoretical predictions [31]).

In the first part of this chapter (after a brief, non-technical theoretical introduction, which can be useful since it informs about an original, still scarcely known approach) we mention the various experimental sectors of physics in which superluminal motions seem to appear. In particular, a bird's-eye view is presented of experiments with evanescent waves (and/or tunnelling photons), and with "localized superluminal solutions" (SLS) to the wave equation, like the so-called X-shaped waves; the shortness of this review is compensated for by a number of references, sufficient in some cases to provide interested readers with reasonable bibliographical information.

2. General Concepts

Let us premise that special relativity (SR), abundantly confirmed by experience, can be built on two simple postulates:

(1) that the laws (of electromagnetism and mechanics) be valid not only for a particular observer, but for the whole class of the "inertial" observers;
(2) that space and time be homogeneous and space be moreover isotropic.

From these postulates one can theoretically *infer* that one, and only one, *invariant* speed exists: and experience tells us such a speed to be the one, c, of light in vacuum (namely, 299,792,458 km/s). Indeed, ordinary light possesses the peculiar feature of always presenting the same speed in vacuum, even when we run towards or away from it. It is just that feature, of being invariant, that makes the speed c quite exceptional: no bradyons, and no tachyons, can enjoy the same property.

Another (known) consequence of our postulates is that the total energy of an ordinary particle increases when its speed v increases, tending to infinity when v tends to c. Therefore, infinite forces would be needed for a bradyon to reach the speed c. This fact generated the popular opinion that speed c can be neither achieved nor overcome.

However, as speed c photons exist which are born, live and die at the speed of light (without any need for acceleration from rest to the light speed), so objects can exist [39] always endowed with speeds V larger than c (see Fig. 1). This circumstance has been picturesquely illustrated by George Sudarshan (1972) with reference to an imaginary demographer studying the population patterns of the Indian subcontinent:

> Suppose a demographer calmly asserts that there are no people North of the Himalayas, since none could climb over the mountain ranges! That would be an absurd conclusion. People of central Asia are born there and live there: they did not have to be born in India and cross the mountain range. So with faster-than-light particles.

Let us add that, still starting from the above two postulates (as well as a third postulate, even more obvious),[3] the theory of relativity can be generalized [31,39] in such a way as to accommodate superluminal objects, a large part of such an extension being contained

[3]Namely, the assumption that particles do not exist – regularly travelling forward in time – endowed with negative energies.

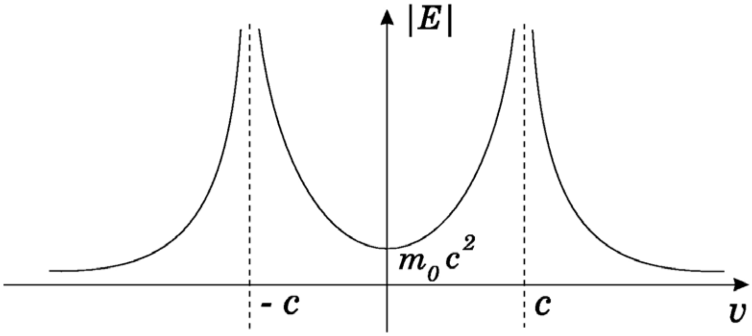

Figure 1. Energy of a free object as a function of its speed [38,31,39].

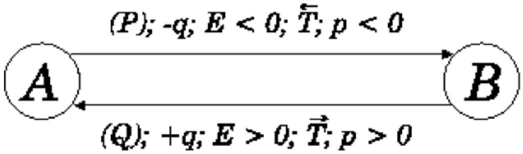

Figure 2. Depicting the "switching rule" (or reinterpretation principle) by Stueckelberg–Feynman–Sudarshan–Recami [31,39,35]: Q will appear as the antiparticle of P. See text.

in a series of works which date back to the 1960s–70s. Also within "extended relativity" [31] the speed c, besides being invariant, is a limiting velocity: but every limiting value has two sides, and one can *a priori* approach it both from the left and from the right.

Actually, the ordinary formulation of SR has been restricted too much. For instance, *even leaving aside superluminal speeds*, it can be easily widened to include antimatter [35]. Then, one finds space–time to be *a priori* populated by normal particles P (which travel forward in time carrying positive energy), *and* by dual particles Q "which travel backwards in time carrying negative energy". The latter shall appear to us as antiparticles, i.e. as particles – regularly travelling forward in time with positive energy, but – with all their "additive" charges (e.g. the electric charge) reversed in sign: see Fig. 2.

To clarify this point, we can recall what follows: We, as macroscopic observers, have to move in time along a single, well-defined direction, to such an extent that we cannot even see a motion backwards in time… and every object like Q, travelling backwards in time (with negative energy), will be *necessarily* reinterpreted by us as an anti-object, with opposite charges but travelling forward in time (with positive energy): cf. Fig. 2 and [31,39,35].

But let us forget about antimatter and go back to tachyons. A strong objection against their existence is based on the opinion that by using tachyons it would be possible to send signals into the past, owing to the fact that a tachyon T which, say, appears to a first observer O as emitted by A and absorbed by B, can appear to a second observer O' as a tachyon T' which travels backwards in time with negative energy. However, by applying (as it is obligatory to do) the same "reinterpretation rule" or switching procedure seen above, T' will appear to the new observer O' just as an antitachyon $\overline{\mathrm{T}}$ emitted by B and absorbed by A, and therefore travelling forward in time, even if in the contrary *space* direction. In such a way, every instance of travel towards the past, and every negative energy, disappears [31,39,35].

Starting from this observation, it is possible to solve [35] the so-called causal paradoxes associated with superluminal motions: paradoxes which are the more instructive and amusing, the more sophisticated they are, but that cannot be re-examined here.[4]

Let us mention here just the following. The reinterpretation principle, according to which signals are carried only by objects which appear to be endowed with positive energy, eliminates any information transfer backwards in time; but this has a price: that of abandoning the ingrained conviction that judgement about what is cause and what is effect is independent of the observer. In fact, in the case examined above, the first observer O considers the event at A to be the cause of the event at B. By contrast, the second observer O' will consider the event at B as causing the event at A. All observers will, however, see the cause happen *before* its effect.

Taking new objects or entities into consideration always forces us to review our prejudices. If we require the phenomena to obey the *law* of (retarded) causality with respect to all observers, then we cannot also demand the *description* "details" of the phenomena to be invariant: namely, we cannot also demand in that case the invariance of the "cause" and "effect" *labels* [38,36].

To illustrate the nature of our difficulties in accepting that e.g. the parts of cause and effect depend on the observer, let us cite an analogous situation that does not imply present-day prejudices:

> For ancient Egyptians, who knew only the Nile and its tributaries, which all flow South to North, the meaning of the word "south" coincided with the one of "upstream", and the meaning of the word "north" coincided with the one of "downstream". When Egyptians discovered the Euphrates, which unfortunately happens to flow North to South, they passed through such a crisis that it is mentioned in the stele of Tuthmosis I, which tells us about *that inverted water that goes downstream (i.e. towards the North) in going upstream* [Csonka, 1970].

In the last century theoretical physics led us in a natural way to suppose the existence of various types of objects: magnetic monopoles, quarks, strings, tachyons, besides black holes: and various sectors of physics could not go on without them, even if the existence of none of them is certain (also because attention has not yet been paid to some links existing among them: e.g. a superluminal electric charge is expected to behave like a magnetic monopole; and a black hole *a priori* can be the source of tachyonic matter). According to Democritus of Abdera, everything that was thinkable without meeting contradictions had to exist somewhere in the unlimited universe. This point of view – which was given the name "totalitarian principle" by M. Gell-Mann – was later expressed by T.H. White in the humorous form "Anything not forbidden is compulsory"...

3. A Glance at the Experimental State-of-the-Art

Extended relativity can allow a better understanding of many aspects of *ordinary* physics, even if tachyons would not exist in our cosmos as asymptotically free objects. As already said, we are dealing with superluminal motions, however, since this topic is coming back into fashion, especially because at least three or four different experimental sectors of physics seem to suggest the possible existence of faster-than-light motions. Our first aim is to put forth in the following some information (mainly bibliographical) about the

[4] Some of them have been proposed by R.C. Tolman, J. Bell, F.A.E. Pirani, J.D. Edmonds and others [31,36].

experimental results obtained in a couple of these different sectors, with a mere mention of the others.

3.1. Neutrinos

A long series of experiments, begun in 1971, seems to show that the square ${m_0}^2$ of the mass m_0 of muon-neutrinos, and more recently of electron–neutrinos too, is negative. This, if confirmed, would mean that (when using a naïve language, commonly adopted) such neutrinos possess an "imaginary mass" and are therefore tachyonic, or mainly tachyonic [31,3]. In extended relativity, the dispersion relation for a free superluminal object becomes[5]

$$\omega^2 - \boldsymbol{k}^2 = -\Omega^2, \quad \text{or} \quad E^2 - \boldsymbol{p}^2 = -m_0^2,$$

and there is *no* need therefore of imaginary masses …

3.2. Galactic Micro-Quasars

As to the *apparent* superluminal expansions observed in the core of quasars [48] and, recently, in so-called galactic microquasars [18], we shall not deal with that problem here, because it is far from the other topics of this chapter: not to mention that for those astronomical observations there exist orthodox interpretations, based on Ref. [40], that – even if "statistically" weak – are accepted by the majority of astrophysicists.[6]

Here, let us mention only that simple geometrical considerations in Minkowski space show that a *single* superluminal light source would appear [31,33]: (i) initially, in the "optical boom" phase (analogous to the acoustic "boom" produced by a plane travelling at a constant supersonic speed), as an intense source which suddenly comes into view; and (ii) would afterwards seem to split into two objects receding one from the other with speed $V > 2c$ (all of this being similar to what is actually observed, according to [18]).

3.3. Evanescent Waves and "Tunnelling Photons"

Within quantum mechanics (and, more accurately, in the *tunnelling* processes), it had been shown that the tunnelling time – firstly evaluated as simple "phase time" and later on calculated through the analysis of the wavepacket behaviour – does not depend on the barrier width in the case of opaque barriers ("Hartman effect") [22]. This implies superluminal and arbitrarily large (group) velocities V inside long enough barriers: see Fig. 3.

Experiments that may verify this prediction by, say, electrons are difficult. Luckily enough, however, the Schroedinger equation in the presence of a potential barrier is mathematically identical to the Helmholtz equation for an electromagnetic wave propagating, for instance, down a metallic waveguide along the x-axis (as shown, e.g., by R. Chiao *et al.* [10]); and a barrier height U bigger than the electron energy E corresponds (for a given wave frequency) to a waveguide of transverse size lower than a cut-

[5] We put $c = 1$, whenever convenient, throughout this chapter.

[6] For a theoretical discussion, see [33].

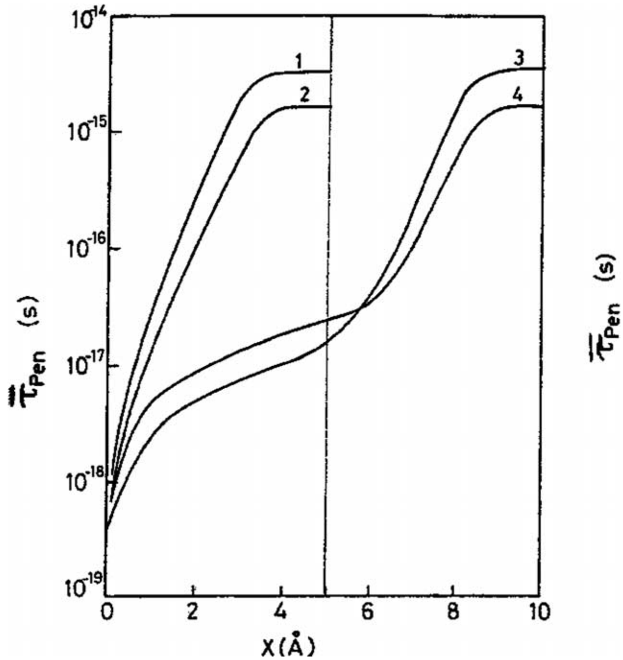

Figure 3. Behaviour of the average "penetration time" (in seconds) spent by a tunnelling wavepacket, as a function of the penetration depth (in åangstroms) down a potential barrier (from Olkhovsky *et al.* [22]). According to the predictions of quantum mechanics, the wavepacket speed inside the barrier increases in an unlimited way for opaque barriers, and the total tunnelling time does *not* depend on the barrier width [22].

off value. A segment of "undersized" guide – to continue with our example – therefore behaves as a barrier for the wave (photonic barrier) [10], as well as any other photonic band-gap filters. The wave assumes therein – like an electron inside a quantum barrier – an imaginary momentum or wave-number and gets, as a consequence, exponentially damped along x. In other words, it becomes an *evanescent* wave (going back to normal propagation, even if with reduced amplitude, when the narrowing ends and the guide returns to its initial transverse size). Thus, a tunnelling experiment can be simulated [10] by having recourse to evanescent waves (for which the concept of group velocity can be properly extended [34]).

The fact that evanescent waves travel with superluminal speeds (cf. e.g. Fig. 4) has actually been verified in a series of famous experiments, performed since 1992 onwards by R. Chiao, P.G. Kwiat and A. Steinberg's group at Berkeley [44], by G. Nimtz *et al.* at Cologne [20], by A. Ranfagni and colleagues at Florence [30], and by others at Vienna, Orsay and Rennes [30], which verified that "tunnelling photons" travel with superluminal group velocities.[7] Let us add also that extended relativity had predicted [50] evanescent waves endowed with faster-than-c speeds; the whole matter therefore appears to be theoretically consistent. The debate in the current literature does not refer to the experimental results (which can be correctly reproduced by numerical elaborations [8,4]

[7] Such experiments also raised a great deal of interest [49] within the non-specialized press, and were reported by *Scientific American*, *Nature*, *New Scientist*, *Newsweek*, etc.

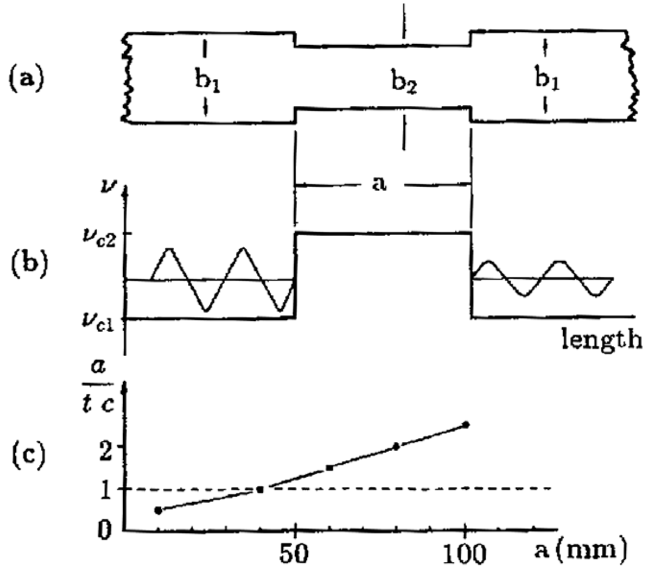

Figure 4. Simulation of tunnelling by experiments with evanescent classical waves (see text), which were predicted to be superluminal on the basis of extended relativity [31,39]. The figure shows one of the measurement results in [44]; that is, the wavepacket average speed while crossing the evanescent region (= segment of undersized waveguide, or "barrier") as a function of its length. As theoretically predicted [22, 50], such an average speed exceeds *c* for long enough "barriers".

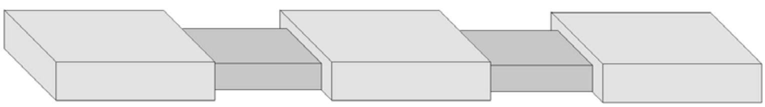

Figure 5. The very interesting experiment along a metallic waveguide with two barriers (undersized guide segments), i.e. with two evanescence regions [21]. See text.

based on Maxwell equations only), but rather to the question whether they allow, or do not allow, the sending of signals or information with superluminal speed [34,4,17].

In the above-mentioned experiments one meets a substantial attenuation of the considered pulses during tunnelling (or during propagation in an absorbing medium). However, by employing a "gain doublet", it has recently been reported the observation of undistorted pulses propagating with superluminal group-velocity with a *small* change in amplitude [46].

Let us emphasize that some of the most interesting experiments of this series seem to be the ones with two "barriers" (e.g. with two gratings in an optical fibre, or with two segments of undersized waveguide separated by a piece of normal-sized waveguide: Fig. 5). For suitable frequency bands – i.e. for "tunnelling" far from resonances – it was found that the total crossing time does not depend on the length of the intermediate (normal) guide: namely, that the wavepacket speed along it is infinite [21,13]. This agrees with what was predicted by quantum mechanics for non-resonant tunnelling through two successive opaque barriers under suitable hypotheses (the tunnelling *phase time*, which depends on the entering energy, has been shown by us to be independent of the distance between the two barriers [24]); something that has been accepted and generalized in

Aharonov *et al.* [24]. Such a prediction has been verified a second time, taking advantage of the circumstance that quite interesting evanescence regions can be constructed in the most varied manners, such as by means of different photonic band-gap materials or gratings (it being possible to use multilayer dielectric mirrors, or semiconductors, to photonic crystals ...). And indeed very recent confirmation came – as already mentioned – from an experiment having recourse to two gratings in an optical fibre [13].

We cannot skip a further topic – which, being delicate, should not appear in a brief overview like this one – since the last experimental contribution to it (performed at Princeton by J. Wang *et al.* [46] and published in *Nature* on 20 July 2000) aroused the interest of the world's press.

Even if in extended relativity all the ordinary causal paradoxes seem to be solvable [31,36], nevertheless one has to bear in mind that (whenever an object, $\mathcal{O}$, travels with superluminal speed) one may have to deal with negative contributions to the *tunnelling times* [22,25]: and this should not be regarded as unphysical. In fact, whenever an "object" (particle, electromagnetic pulse, etc.) $\mathcal{O}$ *overcomes* the infinite speed [31,36] with respect to a certain observer, it will afterwards appear to the same observer as the "*anti*-object" $\overline{\mathcal{O}}$ travelling in the opposite *space* direction [31,36].

For instance, when going on from the lab to a frame $\mathcal{F}$ moving in the *same* direction as the particles or waves entering the barrier region, the object $\mathcal{O}$ penetrating through the final part of the barrier (with almost infinite speed [22,4,24,25], as in Fig. 3) will appear in the frame $\mathcal{F}$ as an anti-object $\overline{\mathcal{O}}$ crossing that portion of the barrier *in the opposite space-direction* [31,36]. In the new frame $\mathcal{F}$, therefore, such an anti-object $\overline{\mathcal{O}}$ would yield a *negative* contribution to the tunnelling time, which could even be negative. For clarifications, see [23]. What we want to stress here is that the appearance of such negative times is predicted by relativity itself, on the basis of the ordinary postulates [31,36,4,23]. (In the case of a non-polarized wave, the wave anti-packet coincides with the initial wave packet; if a photon is, however, endowed with helicity $\lambda = +1$, the anti-photon will bear the opposite helicity $\lambda = -1$.)[8]

Let us add here that, via quantum interference effects it is possible to obtain dielectrics with refraction indices very rapidly varying as a function of frequency, also in three-level atomic systems, with almost complete absence of light absorption (i.e. with quantum induced transparency) [1]. The group-velocity of a light pulse propagating in such a medium can decrease to very low values, either positive or negatives, with no pulse distortion. Experiments have been performed both in atomic samples at room temperature and in Bose–Einstein condensates, which showed the possibility of reducing the speed of light to a few metres per second. Similar, but negative, group velocities, implying a propagation with superluminal speeds thousands of times higher than the previously mentioned ones, have also been recently predicted in the presence of such an "electromagnetically induced transparency", for light moving in a rubidium condensate [2], while corresponding experiments are being carried out, for instance, at the LENS laboratory, Florence.

Finally, let us recall that faster-than-c propagation of light pulses can also be (and was, in same cases) observed by taking advantage of anomalous dispersion near an absorbing line, or nonlinear and linear gain lines – as already seen –, or nondispersive di-

[8]From a theoretical point of view, besides Refs [31,36,22,4,25,23], see Ref. [9]. On the (quite interesting!) experimental side, see papers [47], the first one having already been mentioned above.

electric media, or inverted two-level media, as well as in some parametric processes in nonlinear optics (cf. e.g. G. Kurizki *et al.*'s works).

3.4. Superluminal Localized Solutions (SLS) to the Wave Equations. The "X-Shaped Waves"

The fourth sector is no less important. It came back into fashion when some groups of capable scholars in engineering (for sociological reasons, most physicists had abandoned the field) rediscovered by a series of clever works that any wave equation – to fix the ideas, let us think of the electromagnetic case – also admit solutions as much subluminal as superluminal (besides the ordinary waves endowed with speed c/n).

Let us recall that, starting with the pioneering work by H. Bateman, it had slowly become known that all homogeneous wave equations (in a general sense: scalar, electromagnetic, spinorial, ...) admit wavelet-type solutions with subluminal group velocities [6]. Subsequently, also superluminal solutions started to be written down.[9] An important feature of some of these new solutions (which attracted much attention for possible applications) is that they propagate as localized, non-diffracting pulses: namely, according to Courant and Hilbert's terminology [6], as "undistorted progressive waves". It is easy to realize the practical importance, for instance, of a radio transmission carried out by localized waves, independently of their being sub- or superluminal. But non-diffractive wave packets can be of use even in theoretical physics for a reasonable representation of elementary particles [42]; and so on.

Within extended relativity since 1980 it had been found [5] that – while the simplest subluminal object conceivable is a small sphere, or a point as its limit – the simplest superluminal objects turns out to be instead an "X-shaped" wave (see Refs [5], and Figs 6 and 7 of this chapter), or a double cone as its limit, which moreover travels without deforming – i.e. rigidly – in a homogeneous medium [31]. It is not without meaning that the most interesting localized solutions happened to be superluminal, and with a shape of that kind. Even more, since from Maxwell equations under simple hypotheses one goes on to the usual *scalar* wave equation for each electric or magnetic field component, one can expect the same solutions to exist also in the fields of acoustic waves and of seismic waves (and of gravitational waves too).

Actually, such waves (as suitable superpositions of Bessel beams [12]) were mathematically constructed for the first time, by Lu *et al.* [14], *in acoustics*: and later on by Recami [32] for electromagnetism; they were then called "X-waves" or, rather, X-shaped waves. In the Appendix we briefly show how X-shaped solutions to the wave equation (in particular, the "classical" X-wave) can be constructed.

It is more important for us that the X-shaped waves have indeed been produced in experiments both with acoustic and with electromagnetic waves; that is, X-waves were produced that travel undistorted faster than the speed of sound, in the first case, and than light, in the second case. In acoustics, the first experiment was performed by Lu *et al.* [15] in 1992 at the Mayo Clinic (and their papers received the 1992 first IEEE award). In the electromagnetic case, which is more intriguing, superluminal localized X-shaped solutions were first mathematically constructed (cf. e.g. Fig. 8) in Refs [32], and later experimentally produced by Saari *et al.* [41] in 1997 at Tartu by visible light (Fig. 9),

[9] This was done in [45] and, independently, in [11] (in one case just by the mere application of a superluminal Lorentz "transformation" [31,37]).

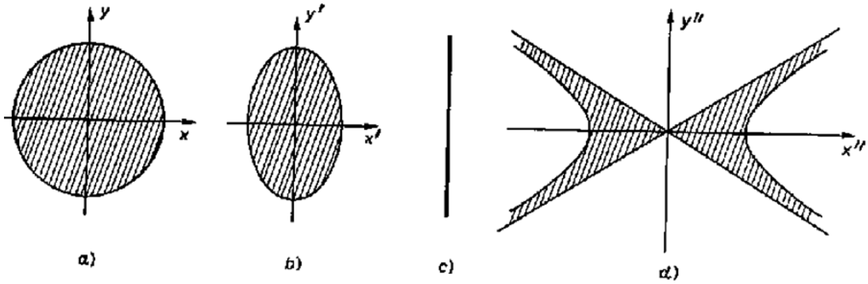

Figure 6. An intrinsically spherical (or pointlike, at the limit) object appears in the vacuum as an ellipsoid contracted along the motion direction when endowed with a speed $v < c$. By contrast, if endowed with a speed $V > c$ (even if the c-speed barrier cannot be crossed, neither from the left nor from the right), it would appear [42] no longer as a particle, but rather as an "X-shaped" wave [42] travelling rigidly (occupying the region delimited by a double cone and a two-sheeted hyperboloid – or as a double cone, at the limit – moving superluminally and without distortion in the vacuum, or in a homogeneous medium).

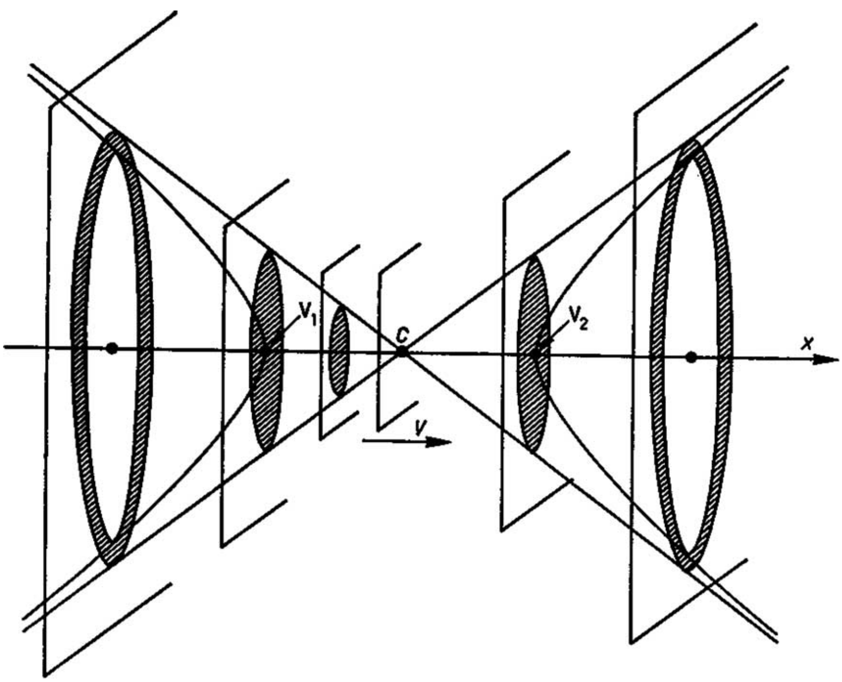

Figure 7. Here we show the intersections of an "X-shaped wave" [42] with planes orthogonal to its motion line, according to extended relativity [38,31,39]. Examination of this figure suggests how to construct a simple dynamic antenna for generating such localized superluminal waves (such an antenna was in fact adopted, independently, by Lu *et al.* [14] for the production of such non-diffractive waves).

and more recently by Mugnai, Ranfagni and Ruggeri at Florence by microwaves [19] (paper appeared in *Phys. Rev. Lett.* of 22 May 2000). Further experimental activity is in progress, while in the theoretical sector the activity has been growing so rapidly that it is not possible to quote here the relevant recent literature; we might recall, e.g., the papers devoted to building up new analogous solutions with finite total energy or more suitable

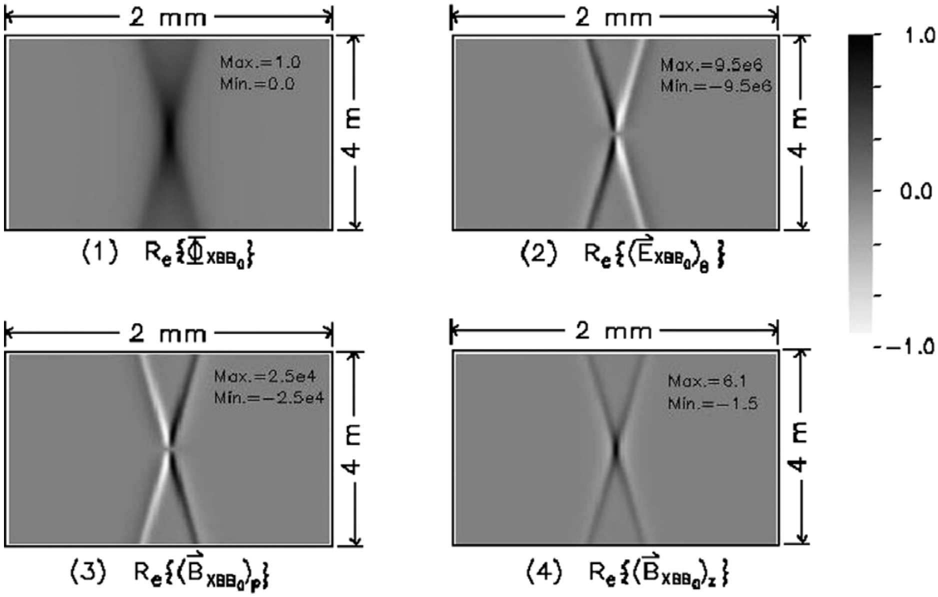

Figure 8. Theoretical prediction of the superluminal localized "X-shaped" waves for the electromagnetic case (from Lu, Greenleaf and Recami [32], and Recami [32]).

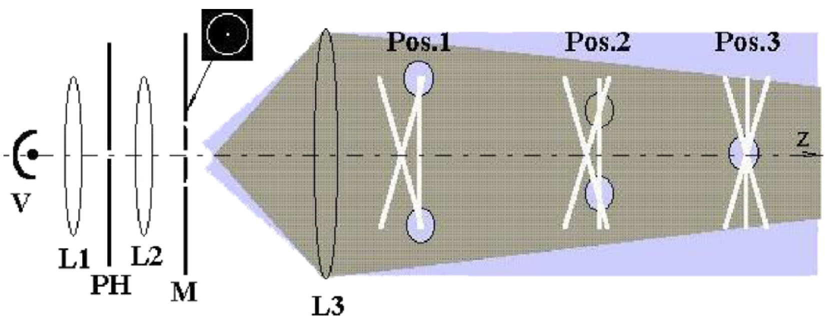

Figure 9. Scheme of the experiment by Saari *et al.*, who announced (*Physical Review Letters* of 24 Nov. 1997) the production in optics of the waves depicted in Fig. 8. In this figure one can see what was shown by the experiment, i.e. that the superluminal "X-shaped" waves run after and catch up with the plane waves (the latter regularly travelling with speed *c*). An analogous experiment has been performed with microwaves at Florence by Mugnai, Ranfagni and Ruggeri (*Physical Review Letters* of 22 May 2000).

for high frequencies, on one hand, and localized solutions superluminally propagating even along a normal waveguide, on the other hand [26,27]; or attempts at focusing X-shaped waves, at a certain instant, in a small region [28]. But we cannot avoid mentioning that suitable superpositions of Bessel beams can produce a *stationary* intense wave-field: confined within a tiny region, and static; while the field intensity outside the tiny region is negligible [29]; such "frozen waves" can have very many important applications, of course (a patent is pending).

Before continuing, let us eventually touch on the problem of producing an X-shaped superluminal wave like the one in Fig. 7, but truncated – of course – in space and in time (by the use of a finite antenna, radiating for a finite time): in such a situation, the wave will keep its localization and superluminality only along a certain "depth of field", decaying abruptly afterwards [12,32].

We can become convinced about the possibility of realizing it, by imagining the simple ideal case of a negligibly sized superluminal source S endowed with speed $V > c$ in a vacuum and emitting electromagnetic waves W (each travelling at the invariant speed c). The electromagnetic waves will be internally tangent to an enveloping cone C having S as its vertex, and as its axis the propagation line x of the source [31].

This is analogous to what happens for a plane that moves in the air with constant supersonic speed. The waves W interfere mostly negatively inside the cone C, and constructively on its surface. We can place a plane detector orthogonally to x, and record the magnitude and direction of the W waves that hit on it, as (cylindrically symmetric) functions of position and of time. It will be enough, then, to replace the plane detector with a plane antenna which *emits* – instead of recording – exactly the same (axially symmetric) space–time pattern of waves W, for constructing a cone-shaped electromagnetic wave C that will propagate with the superluminal speed V (of course, without a source any longer at its vertex): even if each wave W travels at the invariant speed c.[10]

Here let us only remark that such localized superluminal waves appear to keep their good properties only as long as they are fed by the waves arriving (at speed c) from the antenna. Taking account of the time needed for fostering such superluminal pulses (i.e. for the arrival of the feeding speed-c waves coming from the aperture), one concludes that these localized superluminal waves are probably unable to transmit *information* faster than c. However, they don't seem to have anything to do with the illusory "scissors effect", even if the energy feeding them appears to travel with the speed of light. In fact, the spot – endowed, as we know, with superluminal group-velocity – is able to get, for instance, two (tiny) detectors at a distance L to click after a time *smaller* than L/c. A lot of discussion is still going on about the possible differences among group-velocity, signal-velocity and information speed.

As we mentioned above, the existence of all these X-shaped superluminal (or "supersonic") waves seems to constitute at the moment, together, e.g. with the superluminality of evanescent waves, nothing but confirmations of extended relativity: a theory, let us repeat, based on the ordinary postulates of SR and which consequently does not appear to violate any of its fundamental principles. It is curious, moreover, that one of the first applications of such X-waves (which takes advantage of their propagation without deformation) is in progress in the field of medicine; more precisely, ultrasound scanners [16].

Before ending, let us remark that a series of new SLSs to the Maxwell equations, suitable for arbitrary frequencies and arbitrary bandwidths, have recently been constructed by us, some of them being endowed with *finite* total energy. Among the others, we have set forth an infinite family of generalizations of the classical X-shaped wave; and shown how to deal with the case of a *dispersive* medium. Results of this kind may find application in other fields in which an essential role is played by a wave-equation (such as acoustics, seismology, geophysics, gravitation, elementary particle physics, etc.).

[10] For further details, see the first of Refs [32].

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Appendix

In this Appendix we want to show how localized superluminal solutions (SLS) to the wave equations, in particular the X-shaped ones, can be mathematically constructed. Here for simplicity we shall consider only the case of a dispersionless medium such as a vacuum, and of free space (without boundaries).

It has been known for over a century that a particular axially symmetric solution to the wave equation in vacuum ($n = n_0$) is, in cylindrical coordinates, the function $\psi(\rho, z, t) = J_0(k_\rho \rho)\, \mathrm{e}^{+ik_z z}\, \mathrm{e}^{-i\omega t}$ with $k_\rho^2 = n_0^2(\omega^2/c^2) - k_z^2$; $k_\rho^2 \geq 0$, where J_0 is the zeroth-order ordinary *Bessel function*, k_z and k_ρ are the axial and the transverse wavenumbers respectively, ω is the angular frequency and c is the light velocity. Using the transformation

$$\begin{cases} k_\rho = \dfrac{\omega}{c} n_0 \sin\theta, \\ k_z = \dfrac{\omega}{c} n_0 \cos\theta \end{cases} \tag{1}$$

the solution $\psi(\rho, z, t)$ can be rewritten in the well-known *Bessel beam* form:

$$\psi(\rho, \zeta) \;=\; J_0\!\left(n_0 \frac{\omega}{c} \rho \sin\theta\right) \mathrm{e}^{+in_0(\omega/c)\zeta\cos\theta}, \tag{2}$$

where $\zeta \equiv z - Vt$ while $V = c/(n_0 \cos\theta)$ is the phase velocity, quantity θ $(0 < \theta < \pi/2)$ being the cone angle of the Bessel beam.

More generally, SLSs (with axial symmetry) to the wave equation will be the following ones [14,43]:

$$\psi(\rho, \zeta) \;=\; \int_0^\infty S(\omega) J_0\!\left(\frac{\omega}{V} \rho \sqrt{n_0^2 \frac{V^2}{c^2} - 1}\right) \mathrm{e}^{+i(\omega/V)\zeta}\, \mathrm{d}\omega, \tag{3}$$

where $S(\omega)$ is the adopted frequency spectrum.

Indeed, such solutions result in pulses propagating in free space without distortion and with the superluminal velocity $V = c/(n_0 \cos\theta)$. The most popular spectrum $S(\omega)$ is that given by $S(\omega) = \mathrm{e}^{-a\omega}$, which provides the ordinary ("classical") X-shaped wave

$$X \equiv \psi(\rho, \zeta) \;=\; \frac{V}{\sqrt{(aV - i\zeta)^2 + \rho^2(n_0^2(V^2/c^2) - 1)}}. \tag{4}$$

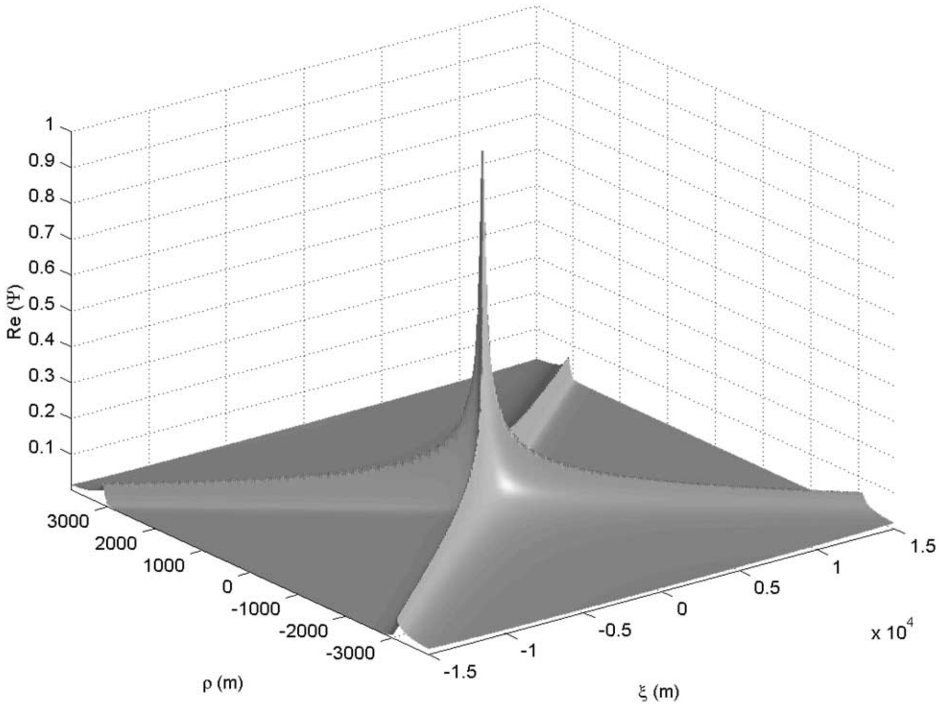

Figure 10. The figure represents (in arbitrary units) the square magnitude of the "classical", X-shaped superluminal localized solution (SLS) to the wave equation, with $V = 5c$ (and $a = 0.1$): see [14,32]. It has been shown elsewhere that (infinite) families of SLSs exist, however, which generalize this classical X-shaped solution.

Because of its non-diffractive properties and its low frequency spectrum,[11] the X-wave is being particularly applied in fields such as acoustics [15]. The "classical" X-wave is represented in Fig. 10.

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