Physics Before and After Einstein
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Chapter 9

The Quantum Debate: From Einstein to Bell and Beyond

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1. Introduction

Einstein's thought processes are very complex ones. Not even a century of diversified research on him and his work has been enough to explore all his epistemological and methodological insights. One might argue that this is the situation for all the great thinkers of humankind: in other words, the thoughts of any great thinker necessarily involve a high degree of complexity. This is true, but in Einstein's case we can assert that the most representative statements attributed to him have been interpreted by the scholars in an inadequate way. So, our feeling is that in spite of the vast literature about Einstein's work, it is necessary to improve the present Einsteinian hermeneutic to overcome the serious problems appearing in the Tower of Babel that has been built up round Einstein's memory.

Consider, for example, Einstein's famous statement according to which "God does not play dice". This is frequently interpreted as the 'conservative' attitude of a great thinker who was unable to overcome the grip of the classical concept of physical reality, despite overwhelming experimental evidence to the contrary. Although it is possible to criticize Einstein, for example, because of the 'super-determinism' of his special theory of relativity, we believe that to qualify him as a 'conservative' constitutes a misunderstanding, which is, at the same time, an epistemological obstacle hindering the comprehension of the depth of his thought. In the present chapter, we wish to document and explain this misunderstanding.

Einstein was not only an extraordinary scientist who marked indelibly the course of the development of physics with his seminal work on relativity and quantum theory. Einstein's intellectual activity was very broad and highly comprehensive, including themes as diversified as the epistemological implications of twentieth-century physics, education, pacifism, freedom, citizenship, the intellectual and political autonomy of the individual, and so on. Einstein is also famous for his ideal of a 'world government', firmly based on justice, peace and prosperity for all people. In a world with apparently insurmountable social, economical, cultural and regional inequalities, and in which harmful, dogmatic and even fundamentalist attitudes play a dominant role, Einstein's dream seems to be somewhat naïve and utopian. But Einstein was aware of the immense difficulties that had to be overcome in order to implement his ideas. He argued that, regrettably, the struggle against one ‘organized power’ necessarily requires another ‘organized power’ (see [30, Chapter 25, Section 25b, p. 539]). However, everyone must play his – or her – role.

It is crucial to emphasize that Einstein’s pacifist attitude was an important aspect of his lucid *rationalism*. In simple and direct terms, Einstein considered war to be barbaric and at the same time the worst way to solve humankind’s complex problems. Peace, critical discussion, freedom of thought and of expression, tolerance with respect to the diversity of opinions (though not tolerance of intolerance), and the autonomy of the individual are no mere marginal part of his rationalism. On the contrary, these aims and values are a central part of it.

In order to better situate Einstein’s *rationalism*, we must connect it strongly to his *realism*. This approach is best adapted to providing the means to confront his scientific research programme with rival ones, principally Bohr’s.

We shall organize this chapter as the study of the confrontation among scientific research programmes. This concept was proposed by Imre Lakatos, in order to account for some important difficulties arising in Popper’s falsificationism. Although Lakatos’ philosophy is chronologically posterior to Einstein’s death in 1955, we think that it is very useful to interpret several points of Einstein’s thinking.

2. The Idea of a Scientific Research Programme

The methodology of scientific research programmes constitutes an attempt due to Lakatos to correct some exaggerations and even inadequacies of the naïve version of Popper’s falsificationism. According to Popper, the truth of a given scientific theory cannot be definitively proved, no matter how much the theory has been empirically corroborated. In other words, even in cases of extensively and broadly corroborated theories, the truth of these theories cannot be taken for granted. In fact, the existence of an empirical refutation invalidating these theories is always conceivable. On the other hand, one can provide proof that a theory is false: just one counterexample is enough. According to Popper, the method of science consists of bold *conjectures* followed by severe attempts at *refutation*, and in this process the critical discussion and intellectual honesty of the scientists involved play a central role.

Several authors[1] have criticized naïve falsificationism by arguing that the eventual falsehood of a given theory also cannot be conclusively proven, because the observational statements which constitute the basis of the refutation may be revealed to be false in the light of future developments.

Lakatos also criticized Popper’s views. According to Lakatos, naive falsificationism does not correspond to the real development of science. Lakatos partially adopts the rationalistic commitment of Popper according to which we must not put up with contradictions. Thus Popper and Lakatos agree in their struggle against Hegel, who raised contradiction to the category of a supreme virtue. Although supporting Popper in his criticism of Hegelian irrationalism, Lakatos disagrees with him as regards refutation on the basis of only one counterexample: scientists do not abandon their theories when a counterexample arises. Lakatos asserts that it is also possible to progress in science by working

[1] See [14] for an outline of the debate.

on the basis of inconsistent provisional foundations. For example, Bohr's atomic theory contradicts classical electrodynamics, but in spite of this fact it allowed an important advance in science, in the context of a rational scientific programme in which the *correspondence principle* plays a central role.

Lakatos' methodology of scientific research programmes can be outlined as follows. A given scientific theory should be seen as the union of a *hard core* and a *protective belt*. The *hard core* is the unchangeable part of the theory, so considered by methodological decision of adherents of the theory. The *protective belt*, on the other hand, is the changeable part, which may be variously modified and adjusted during the development of the theory. When this accommodation process leads to new possibilities and results, it is said that the adherents of the theory are working in the context of a positive heuristic. Otherwise, when the accommodation process hinders the development of the theory, by making regressive steps and stumbling on insurmountable difficulties, then it is said that the adherents of the theory are working in the context of a regressive or negative heuristic.

This flexibility allows for greater freedom of investigation because one empirical test is normally unable to invalidate a great idea. There is a very complex dialogue between theory and experiment, necessarily accompanied by an equally complex confrontation among webs of theories. In short, all experimental tests involve a confrontation of webs of theories. The concept of an *objective reality* is preserved, but one can decide whether an experimental test is "crucial" only with reference to an accepted theoretical framework. Of course, this limitation characterizes any theory, and cannot be interpreted as an argument in favour of the thesis of the "dissolution" of reality, or other instrumentalist and positivist claims. It is essential to emphasize that the existence of reality does not constitute a result, but a starting point, and as a consequence it is impossible to prove the existence of reality. Realism is a postulate, so it cannot be refuted by any experiment. Of course, the same holds for the opposite philosophical view.

3. Einstein's Scientific Research Programme

Physics can be considered a web of diversified scientific research programmes. This statement has nothing to do with cultural relativism. On the contrary, our starting points are: (1) the existence of an objective reality independent of ourselves and of our desires (realistic adoption), and (2) the philosophical assumption that the world is comprehensible, or in other words, that human reason plays a central role in the understanding of the world (rationalist adoption). We emphasize here that the term *rationalism* must be interpreted in a very broad sense. It differs from seventeenth-century rationalism in that it comprehends both rationalism and empiricism, as entangled intellectual approaches to knowledge. In this broad sense, the epistemologies due to Bachelard, Popper and Lakatos, for example, are all rationalist.

The statement that physics consists of an entangled web of diversified scientific research programmes might lead to the conclusion that instrumentalist and positivist approaches would be equally acceptable. But we wish to emphasize that this is not the case. Instrumentalist and positivist approaches to reality contain an explicit cognitive renunciation that hinders the search for knowledge in depth.

The web of science is embedded inside a rationalistic and realistic attitude, admitting both the complexity of human thought and the complexity of reality. But it is important to say that in spite of scientific theories being human constructions, this does not undermine the autonomy of reality itself.

What is the Einsteinian Scientific Research Programme?

The answer to this question is not easy, given the great complexity of Einstein's thought, but fortunately we can outline Einstein's research objectives here. Einstein's ontology is clear: physical theories are about what things are and not about what we can say about what things are. Einstein does not confuse ontology with epistemology.

Another important point of his research programme can be expressed by one of his famous statements: "The world of our sense experiences is comprehensible. The fact that it is comprehensible is a miracle".[2] Sometimes this statement appears in an alternative form: "The most incomprehensible thing about the world is that it is comprehensible".[3]

Let us give one provisional statement of the *hard core* of the realistic and rationalistic Einsteinian research programme:

> $\mathbf{E_1}$: *Physical theories are attempts at saying how things are. The world is comprehensible.*

Clearly the above statement is a very general one. We need to say something else about the term *comprehensibility*. Of course, the statement $\mathbf{E_1}$ seems to be not enough to characterize uniquely Einstein's programme. In fact, $\mathbf{E_1}$ is also perfectly adaptable to the Galilean, Cartesian, Newtonian, Leibnizian, Maxwellian and several other scientific programmes. However, according to Einstein, quantum objects are concrete entities existing in a space–time where causality holds. Thus, in order to express Einstein's thought we ought to enrich $\mathbf{E_1}$. Our second tentative statement is:

> $\mathbf{E_2}$: *Physical theories (including quantum theory) are attempts at saying how things are (including quantum objects). The objective world (including the quantum world) is comprehensible. By the simultaneous help of space–time and causal conceptual categories we can study this comprehensible world.*

$\mathbf{E_2}$ is more precise than $\mathbf{E_1}$, and exhibits the peculiarity of Einstein's approach. To better characterize this peculiarity we will compare Einstein's and Niels Bohr's scientific research programmes.

To make explicit Einstein's claims in favour of objectivity and independence of reality we end this section by quoting his reality criterion stated in his famous EPR paper:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity [16].

4. The Debate on Quantum Mechanics: A Chronological Outline

A precise and complete chronological outline of the debate on quantum mechanics is beyond the scope of this chapter. The literature on the quantum mechanics debate is enormous. This debate has been studied from various point of views by philosophers, scientists, scientists with philosophical inclinations, intellectual people of several inclinations

[2] Einstein [18, p. 292] *apud* Lindley [27, p. 4].
[3] Einstein *apud* Gal-Or [21, p. 166].

and so on. However, it is possible to offer a rough outline of some important events that marked the history of quantum theory.

First, we can say that quantum theory constitutes a collective construction of one hundred scientists (perhaps two or three hundred). In a relatively recent essay [36,38], a well-known living scholar made one difficult choice by proposing a list of 12 principal names. However, this choice is so restrictive, and excludes several important contributors to the theory. The list covers three generations of physicists divided respectively, into an old generation, an intermediate generation and a young generation. All of these 12 physicists are dead. Those belonging to (1) the old generation are Max Planck (1858–1947) and Arnold Sommerfeld (1868–1951); (2) the intermediate generation are Albert Einstein (1879–1955), Paul Ehrenfest (1880–1933), Max Born (1882–1970), Niels Bohr (1885–1962) and Erwin Schrödinger (1887–1961); and (3) the young generation are Louis de Broglie (1892–1987), Wolfgang Pauli (1900–1958), Werner Heisenberg (1901–1976), Pascual Jordan (1902–1980) and Paul Dirac (1902–1984).

It is widely accepted that the construction of quantum mechanics can be considered as taking place in two principal periods: (i) 'old' quantum mechanics (1900–1924) and, (ii) new and orthodox quantum mechanics (1924–1927).

Roughly speaking, in the first period a provisional quantum theory was constructed in which the classical tradition of physics was combined with the postulates of quantization of energy and angular momentum of Pythagorean inspiration. Very important results were achieved, such as the derivation of the black-body radiation formula by Planck (1900), a new and original explanation for the photoelectric effect by Einstein (1905), the formulation of Bohr's theory of the atom (1913) and several others. In spite of this fact and of a clear awareness of the beginning of a revolutionary period, with the creation of the quantum and relativistic theories, no claim was made for a radical reappraisal of microscopic reality. The scientific community conceived that physics could be understood on the basis of the same general principles consolidated by the classical tradition since the days of Galileo and Newton. According to this tradition, roughly speaking, phenomena were considered as taking place in an objective space–time and as ruled by causal laws, principally the conservation laws.

The second period, however, constituted a radical change. Over a very short time (1924–1927), a debate was launched on the problems of the wave–particle duality, leading to the interpretation of Heisenberg's theorem, to the physical interpretation of the Ψ-function (the solution of Schrödinger's equation), and to Born's statistical interpretation of the wave function. As a result, a new conception of the micro-world was adopted under the leadership of Niels Bohr.

The physicists of the Copenhagen School of thought (Bohr, Heisenberg and Pauli)[4] claimed that a radical change of view on microscopic reality was necessary: nature could not be properly understood by adopting the conception of reality consolidated by classical physics. The new conception grew to dominate physics. But a highly qualified minority, including Einstein, de Broglie, Ehrenfest, Schrödinger, Planck and von Laue, disagreed with it.

The central debate, broadly speaking, was a confrontation between Bohr and Einstein, who considered the Copenhagen interpretation to be an extravagant idea with harm-

[4] Or rather the Copenhagen-Göttingen School (Bohr, Heisenberg, Pauli and Max Born).

ful consequences. In 1927, during an important conference held in Como, Italy, in honour of Alessandro Volta, Bohr explained his *Complementarity Principle*.

In 1935 Einstein, Podolsky and Rosen published a famous paper arguing the incompleteness of quantum theory. The paper received, after a few months, a reply from Bohr himself.

In 1964, John Stuart Bell made very important progress with his famous inequality which offered a criterion, at least in principle, to decide experimentally whether Einstein or Bohr was right.

In recent years, the debate has been revisited by physicists and several experiments have been carried out, among them the famous Aspect's experiments in 1982. In spite of widespread opinion that these experiments decided in favour of Bohr's conception, including a very strange non-locality property of quantum reality, there has been much qualified criticism of this hasty conclusion.

Over the last fifteen years there have been several international conferences on the foundations of quantum theory; I wish to mention here at least the International Conference on Bell's Theorem and Foundations of Modern Physics (Cesena, Italy, 1991) [44] and the International Conference on the Frontiers of Fundamental Physics (Olympia, Greece, 1993) [1,2].

5. Niels Bohr's Scientific Research Programme

Niels Bohr's scientific research programme can be summarized in the following way:

> $\mathbf{B_1}$: *Classical theories are attempts at saying how things are. The objective classical world is comprehensible. By using both space–time and causal categories we can study the classical world, but* not *the quantum world.*

Let us give a quotation from Bohr himself:

> $\mathbf{B_2}$: *"I advocated a point of view conveniently termed "complementarity", suited to embrace the characteristic features of individuality of quantum phenomena, and at the same time to clarify the peculiar aspects of the observational problem in this field of experience. For this purpose, it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms"[10, p. 39].*

According to Bohr, quantum phenomena are not comprehensible in the same sense as classical phenomena. In classical physics, objects are spatial and temporal entities that are ruled by causal laws. In this way, classical phenomena are comprehensible according to causal laws (conservation laws) in space–time. Thus in a classical context, the conceptual categories of *space–time* and *cause* can be used together to study physical phenomena.

However, according to Bohr the situation changes drastically when we are dealing with quantum phenomena. In microphysics, the categories of space–time and cause can be used only in a mutually exclusive manner, according to Bohr's *complementarity principle*. Bohr argued in favour of the "indispensable use of classical concepts [...] even though classical physical theories do not suffice" [9, p. 701]. This is how Henry Stapp described Bohr's programme:

> According to Niels Bohr, quantum theory must be interpreted, not as a description of nature itself, but merely as a tool for making predictions about observations appearing under conditions described by classical physics [41, p. 255].

In other words, although quantum phenomena cannot be described by simultaneous use of the space–time and causal concepts, the use of these and other classical physics concepts is unavoidable. If in a given experiment the space–time aspect is implemented, then the causal aspect disappears, and vice versa. In quantum mechanics the causal and space–time categories are mutually exclusive but complementary.

The same holds for particle and wave categories of classical physics: in experiments where corpuscular aspects are implemented, causal aspects are excluded; analogously, in the context of experiments in which wave aspects are implemented, corpuscular aspects are excluded. Once more, they are mutually exclusive categories, but they are complementary aspects in the context of mathematical formalism.

6. Einstein and Objective Reality

Let us come back to Einstein. In order to better outline his beliefs, it is necessary to take into account some aspects of his struggle against the adherents of the so-called "dissolution of reality" thesis, which he considered one of the most harmful and regrettable things for the development of science, for education in the broad sense of the term, and for the intellectual development of individuals. In a famous letter written in April 1938 to his friend Maurice Solovine Einstein wrote about his disappointment:

> In Mach's time a dogmatic materialistic point of view exerted a harmful influence over everything; in the same way today, the subjective and positivistic point of view exerts too strong an influence. The necessity of conceiving of nature as an objective reality is said to be superannuated prejudice while the quanta theoreticians are vaunted. Men are even more susceptible to suggestion than horses, and each period is dominated by a mood, with the result that most men fail to see the tyrant who rules over them [20, p. 85].

The attitude of considering objective reality as a "superannuated prejudice" was completely unacceptable to Einstein. He considered this attitude to be the effect of brainwashing and thus incompatible with genuine realist and rationalist commitments.

Einstein's reaction to the "dissolution of reality" thesis gives rise to one of the most dangerous and perverse misunderstandings in the history of the interpretations of his thought. In spite of his genius and the revolutionary character of his contributions, Einstein is frequently remembered for his supposedly "conservative" attitude firmly based on classical ideas and his "old" concept of reality. Einstein was discontented with this situation. In 1938 he wrote in confidence to Solovine, in a letter where he revealed his plan to defeat what he considered harmful tendencies in physics:

> I am working with my young people on an extremely interesting theory with which I hope to defeat modern proponents of mysticism and probability and their aversion to the notion of reality in the domain of physics. But say nothing about it, for I still do not know whether the end is in sight [20, p. 91].

To accuse Einstein of conservatism because of his concept of physical reality is a mistake. In order to clarify this important point let us consider his theory of the photoelectric effect.

Einstein explained why Maxwell's electromagnetic theory was unable to explain the photoelectric phenomenon: in this phenomenon the central role is played by the *frequency*, not the *intensity*, of light. For this reason we must go beyond Maxwell's electromagnetism. In order to solve this important problem he proposed a new and completely revolutionary idea according to which a spatially concentrated entity carrying the energy $E = h\nu$, where h denotes Planck's constant and ν the frequency of the associated wave, is entirely destroyed in the process, but its energy is entirely conserved, being utilized for: (i) transferring a given energy to an electron of the metallic cathode in order to overcome the potential barrier of the metal, and (ii) for transferring a given kinetic energy to the released electron. Einstein's simple formula is:

$$h\nu = \Phi + E_{\text{kin}},$$

where Φ denotes the energy required to overcome the potential barrier of the metal that constitutes the cathode and E_{kin} denotes the kinetic energy acquired by the released electron.

In 1905, this idea was highly revolutionary. Nobody knew about the existence of a microscopic entity that during the effect is totally destroyed while its energy is conserved. Physicists did not know of anything like this. It is important to emphasize that classical objects such as particles and waves were radically different from this completely new object introduced by Einstein in this highly revolutionary conjecture. Thus, Einstein introduced in physics *the first formulation of particle–wave duality*. Classical physics did not know anything about duality. Particles were dimensionless entities like the points of Euclidean geometry, and waves were spatial and temporal entities characterized by their corresponding frequencies and wavelengths.

Einstein can be considered a precursor of the extraordinary phenomena featuring the annihilation and creation of particles in the high-energy range. A microscopic entity carrying the energy $E = h\nu$ being completely destroyed while its energy is conserved anticipates by a few decades these important phenomena in the high-energy range. Today these phenomena are well known, but in 1905 they were unknown and hard to understand.

It is strange to call 'conservative' the physicist who anticipated the annihilation and creation phenomena, gave the first formulation of the wave–particle duality, and introduced an entirely new object in physics. It is still more surprising, and ironic, that his "old" conception of reality is blamed for his "conservatism"!

But this is not the end of the history. Einstein's papers on relativity, his doctoral thesis and his seminal contribution to the theory of Brownian motion, all of them published during Einstein's *annus mirabilis*, showed, without ambiguity, the strong and broad revolutionary character of his work. In particular, he was the one who definitively established the atomic paradigm in physics, a highly revolutionary event in itself. Thus the epithet of 'conservative' frequently attributed to Einstein cannot be justified.

Einstein knew that a world of intense *becoming*, characterized by the annihilations and creations of the new physics, cannot be understood by extravagant philosophical views such as the "dissolution of reality" thesis. The reality is extremely complex, but this complexity does not mean absurdity. For him the *becoming* shown in high-energy physics experiments co-exists with the *being* guaranteed by conservation laws. Thus the two seminal, although antithetical, philosophical programmes, the Heraclitean and the Parmenidean ones, are both fertile and co-exist in physics. Einstein's strong belief in the objective character of reality can explain his irritation with the adherents of the idealistic mood. The comparison made by him of these adherents with "horses" seems rude, but it arose from a legitimate and constructive irritation. Einstein was seriously worried by the low intellectual standard of the majority of members of the scientific community, who seemed to him highly vulnerable to manipulation and brainwashing.

Let us consider something related to Einstein's famous aphorism according to which "God does not play dice". In a letter dated 30 November 1926 Max Born wrote to Einstein:

> My idea to consider Schrödinger's wavefield as a "Gespensterfeld" in your sense of the word proves to be more useful all the time [. . .]. The probability field propagates, of course, not in ordinary space but in phase space (or configuration space).[5]

Einstein's answer shows his disappointment with the constantly increasing level of abstraction of the quantum formalism, with no intuitive counterpart. Einstein wrote to Born on 4 December 1926:

> Quantum mechanics is very impressive. But an inner voice tells me that is not yet the real thing. The theory produces a good deal but hardly brings as closer to the secret of the Old One. I am at all events convinced that He does not play dice. Waves in 3-n dimensional space whose velocity is regulated by potential energy (e.g. rubber bands) [. . .][6]

The ideas inspired by Einstein's *Führungfeld*, by the inherent duality of all quantum objects due to de Broglie and by the concept of a probability field propagating in a mathematical configuration space, led to a great degree of mathematical abstraction without any corresponding intuitive counterpart. This situation has constituted a serious obstacle which hindered a clear comprehension of how these quantum objects interact in space–time and how they are governed by causal laws.

We must stress that mathematical abstraction in itself is not against causal and space–time explanation. General relativity, for example, involves a high degree of mathematical abstraction and, no doubt, constitutes a causal and space–time theory. With respect to quantum mechanics, mathematical abstraction is accompanied by almost an absence of a physical intuitive counterpart and also by serious ambiguities of interpretation.

Einstein's philosophical position cannot be considered conservative: he had strong motives to deny the new interpretation due to Born.

7. An Interlude: Physics as a Complex Web of Entangled Scientific Research Programmes

Physics is a science that can be considered an entangled and complex web of diversified scientific research programmes. For example, the Parmenidean and Heraclitean programmes are both present in physics today; both historically played a very important role in the development of this science. However, the complexity of physical science goes beyond these two important programmes. We can also assert that the atomistic programme as well as the programme based on continuous conceptual categories such as

[5] Born cited *apud* [30, Chap. 25, p. 526].

[6] Einstein *apud* [30, Chap. 25, p. 527].

plenum, *ether* and *field* have both been enormously fertile for the progress of scientific development, especially physics.

Several other programmes are equally important: the Cartesian and Leibnizian programmes based on conservation laws; the Newtonian programme based on point masses acting at a distance through absolute time and space; the Faraday–Maxwell programme based on continuous electromagnetic waves propagating through the ether at the velocity of light; Einstein's programme, Bohr's programme, the Pythagorean programme, the geometric programme and several others.

The *hard core* of the Pythagorean research programme is the idea that the essence of reality is mirrored in whole numbers and their ratios. In short, according to Pythagoreans, numbers rule all things. However, the Pythagoreans stumbled on a very great difficulty which was, at that time, an insurmountable contradiction and constituted a traumatic discovery, with serious consequences for their programme. They discovered the incommensurability of the diagonal of the square and its side, i.e. that the ratio (d/a), where d denotes the diagonal of the square and a denotes its side, cannot be written as a ratio of whole numbers. In fact, if we allow that the ratio (d/a) is a rational number, then this can be shown to lead to a contradiction according to which a certain integer is both even and odd. *The unavoidable conclusion is that rational numbers do not cover all of conceptual reality*.

It is essential to emphasize that the incommensurability problem appearing in the context of arithmetic is not itself a geometric problem. This circumstance played a central role in the context of the history of the competition between the arithmetic and the geometric programmes; the reason for this can be easily understood. For example, when Socrates (the main character of Plato's dialogue *Meno*) asks the slave boy how long is the side of a square having twice the area of another given square, the answer is that the side of the square having a double area is equal to the diagonal of the other square. The exact solution can be drawn on the earth (or on the sand) without ambiguity and without contradiction. Therefore, the incommensurability problem does not arise geometrically.

Plato also noted this extremely important fact and therefore rated geometry as superior to arithmetic as a world-view. This was, according to Popper, the Platonic and Euclidean Programme (see [33, Chap. 2]). Classical mechanics and the general theory of relativity are frequently considered as emblematic examples of the extraordinary success of the geometric programme in the history of physics.

In spite of the this traumatic affair, the Pythagorean programme resurrected from its ashes – like the phoenix – and played an important end even decisive role in several developments of physics. The branches in which the Pythagorean central idea had most success were electrolysis, the physics of oscillations (normal modes of vibration, as in a vibrating string) and quantum mechanics.

Einstein himself, in his *Autobiographical Notes*, commented in Pythagorean terms about the extraordinary Pythagorean realization of Bohr's theory of the atom. Einstein said that Bohr's theory was "the highest form of musicality in the sphere of thought". The emphasis on music comes, no doubt, as a reference to the Pythagorean Programme. The quantum energy levels enumerated by $n = 1, 2, 3, \ldots$, corresponding to the stationary states of the atom whose transitions occur by 'quantum jumps', i.e. in discrete quantities (emission of photons of well-defined frequencies) are pure Pythagorean music. In the forthcoming section I shall argue that Bohr, Einstein and de Broglie were partially committed to aspects of the arithmetic Pythagorean Programme.

8. Einstein, Bohr and de Broglie

In this section we can see how the Pythagorean programme worked in the context of old quantum mechanics. It is often emphasized that Bohr's theory (1913) contradicts classical electrodynamics. Bohr supposed that electrons perform stationary circular orbits around their nuclei, each with a given constant value of energy. On the other hand, classical electrodynamics holds that an electric charge in circular motion around the nucleus undergoes the action of the centripetal force and, as a consequence, loses its energy and falls into the nucleus. This means that the supposed stability of the atom in Bohr's theory is very strange from the viewpoint of classical electrodynamics.

In order to surmount this paradox, one may attempt to bring into Bohr theory the de Broglie relation

$$p = \frac{h}{\lambda} \tag{1}$$

where p denotes the linear momentum of the particle (an electron in this case), h is the Planck constant and λ is the wavelength of the associated de Broglie wave. As we know, if we consider the nucleus as being at rest (an approximation in which the motion of the nucleus can be neglected due to the fact that the mass of the nucleus is much bigger than the mass of the electron), the total energy of the hydrogen atom (kinetic plus potential energy) is given by

$$E = \frac{p^2}{2m} - \frac{e^2}{r}, \tag{2}$$

where p and m are, respectively, the linear momentum and mass of the electron charge and r is the radius of the supposed circular motion.

If we assume the periodicity condition

$$\lambda = 2\pi r, \tag{3}$$

then the combination of Eqs (1), (2) and (3) gives rise to

$$E = \frac{h^2}{8\pi^2 m r^2} - \frac{e^2}{r}. \tag{4}$$

In order to find the value of r compatible with the minimum value of energy of the hydrogen atom we must derive Eq. (4) with respect to r according to the requirement of minimum energy

$$\left(\frac{\mathrm{d}E}{\mathrm{d}r}\right)_{r=a} = 0. \tag{5}$$

By performing the calculations we arrive at the result

$$a = a_{\mathrm{Bohr}} = \frac{h^2}{4\pi^2 m e^2}. \tag{6}$$

Replacing the value of r in Eq. (4) by the value of a_{Bohr} given by Eq. (6) we obtain the value of the ground state of the hydrogen atom

$$E = E_{\mathrm{Bohr}} = -\frac{2\pi^2 me^4}{h^2}. \tag{7}$$

This procedure can be easily generalized to all stationary states by assuming the relation

$$n\lambda = 2\pi r, \tag{8}$$

where $n = 1, 2, 3, \ldots$. In an analogous way, by combining Eqs (1), (2) and (8) we obtain:

$$E_n = \frac{n^2 h^2}{4\pi^2 2mr^2} - \frac{e^2}{r}. \tag{9}$$

The condition

$$\left(\frac{\mathrm{d}E}{\mathrm{d}r}\right)_{r=a(n)} = 0 \tag{10}$$

leads to a set of values $\{a(n)\}_n$ with $n = 1, 2, 3, \ldots$, i.e.

$$\{a(n)\}_n = \frac{n^2 h^2}{4\pi^2 me^2}. \tag{11}$$

Replacing the value of r in Eq. (9) by the values of $\{a_{\min}\}_n$ given by Eq. (11) and E_n in (9) by $\{E_{\min}\}_n$ we obtain

$$\{E_{\min}\}_n = -\frac{|E_{\mathrm{Bohr}}|}{n^2}. \tag{12}$$

It is important to note that Eqs (11) and (12) give rise, respectively, to Eqs (6) and (7) in the case where $n = 1$, i.e. where a_{Bohr} is the radius *minimum minimorum* and E_{Bohr} is the energy *minimum minimorum* among all the stationary states of the hydrogen atom.

The point to be emphasized here is that even though in the context of a simple theory like this one, Bohr's theory of 1913, the de Broglie relation (Eq. (1)) of 1923–4 constitutes an important explanatory principle providing a better understanding of the mysteries of existence of these stationary states, as well as a possible overcoming of the contradiction between Bohr's theory and the theoretical framework of classical electrodynamics. Surely the de Broglie relation is a general quantum law expressing the inherent objective duality of the quantum objects. When (1) is associated with the periodicity condition expressed by (8) and the minimum condition expressed by (10), it leads, in a natural way, to the stationary states of the hydrogen atom. This means that the understanding of the existence of stationary states of an atom goes beyond the theoretical framework of classical electrodynamics, like the way in which the Planck–Einstein dualistic relation

$$E = h\nu, \tag{13}$$

which is absolutely necessary to explain the photoelectric effect, goes beyond Maxwellian electromagnetism. It is interesting to note that Eqs (1) and (13) are 'twin brothers' expressing the inherent duality of the quantum objects. These relations are touchstones of the quantum theory independent of any particular interpretation.

9. Aspects of the Quantum Debate

Ontology has to do with what kind of things there are, epistemology with our knowledge of things. Of course, our knowledge of the things is different from what the things are. Any objective theory of reality ought to presuppose something about what kind of things there are, independent of our knowing them.

Bohr raised an objection. According to him, it is impossible to separate the things (i.e. the objects of quantum theory) and their interaction with the measuring instruments acting on these things (objects). He argued in favour of

> [...] *the impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear* ([10, pp. 39–40]. The italics are Bohr's).

Bohr's principal reason for defending this "impossibility" depended on the supposed "uncontrollability of the interactions between objects and measurement instruments". He wrote:

> Notwithstanding all novelty of approach, causal description is upheld in relativity theory within any given frame of reference, but in quantum theory the uncontrollable interaction between the objects and the measuring instruments forces us to a renunciation even in such respect [10, p. 41].

According to him, the reason for the renunciation of a causal description in quantum theory concerns the indivisibility of the universal quantum of action expressed by Planck's constant h. This influential idea of Bohr's was combined with the Heisenberg's argument that "in the range of microphysics the ontology of classical physics does not work". Concerning this point, the meaning of the term *phenomenon* according to Bohr is radically different from any other meaning that can be given to this word when the quantum entities are objectively considered as existing, even independently of any measurements. On this important point, Bohr wrote:

> As a more appropriate way of expression I advocated the application of the word *phenomenon* exclusively to refer to the observations obtained under specified circumstances, including an account of the whole experimental arrangement [10, p. 64].

Bohr also argued that this attitude does not imply an arbitrary renunciation, but a clear recognition of the impossibility of a more detailed analysis of atomic phenomena:

> [...] in quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena, but with a recognition that such an analysis is *in principle* excluded. ([10, p. 62]. Cf. Stapp's statement quoted in Section 5.)

Because of Bohr's renunciation of a quantum ontology, the orthodox interpretation of quantum theory he proposed introduced a new methodological choice, which required that (1) quantum reality has to be construed using the classical categories; and (2) in the quantum world, classical categories are mutually exclusive.

His famous reply to Einstein was summarized in the following final passage:

> In fact, it is only the mutual exclusion of two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena that the notion of *complementarity* aims at characterizing [9] (in [10, p. 61]).

This methodological choice constituted an instrumentalist approach to physical science, the principal implication being a programmatic renunciation to presupposing a given quantum reality independent of any observers. Only relevant are "the observations obtained under specified circumstances, including an account of the whole experimental arrangement". According to Bohr the only possible quantum reality is that which is explicitly referred to a macroscopic description of the experimental conditions. In other words, in the domain of quantum reality any statement about these objects, without specifying the conditions under which the observations are made, is meaningless.

Einstein's opposition to this view was not based on pure logic. Einstein argued that the quantum-mechanical description accounts for the behaviour of a large number of atomic systems (an *ensemble*). But his intuition (his inner voice) led him in another direction: he thought that an exhaustive theoretical description of individual phenomena had to be developed. Although Einstein considered that from a strictly logical point of view Bohr's arguments were acceptable, they were not acceptable to his "scientific instinct". In a paper published in 1936, he wrote:

> To believe this is logically possible without contradiction; but it is so very contrary to my scientific instinct that I cannot forego the search for a more complete conception [17] (in [10, p. 61]).

Several "impossibility theorems" were proposed in order to affirm Bohr's point of view in physics. The most famous of these was due to von Neumann, which worked as an epistemological obstacle hindering the search for a causal completion of quantum mechanics. But a simple concrete model of spin-$1/2$ particles and spin measurements [5,36–38] was enough to destroy this presumed impossibility. As Selleri wrote:

> This model reproduces all the quantum mechanical prediction for spin measurements (experimental results and probabilities) and therefore does exactly what von Neumann's theorem attempts to forbid, i.e., provide a causal completion of quantum mechanics [40, p. 48].

David Bohm's famous papers of 1952 played an extremely important role in overcoming the epistemological barrier established by von Neumann's authoritative impossibility theorem. By commenting on Bohm's papers in a humorous and ironical way, Bell declared in 1981: "In 1952 I saw the impossible done" [6].

In spite of the well-known explanation of the existence of stationary states in the context of orthodox quantum mechanics (1927–1930), Bohm's theory can be considered as another possibility of solution of the contradiction between classical electrodynamics and Bohr's atom theory (Appendix A). We remember that according to electrodynamics electrons moving in a circular motion around a nucleus undergo centripetal force and as a consequence of this must lose their energy and fall into the nucleus. For this reason Bohr's atom theory cannot explain the existence of stationary states; consequently, it is also unable to explain, in a completely satisfactory way, the stability of the atom: Bohr's atom theory just *postulates* – as a starting point – the existence of such states. In spite of this, Bohr's theory can calculate stationary states of energy (see Section 8, Eq. (12)).

In 1952, much criticism was levelled at Bohm's ideas. His last statement on the subject is in the book written with Basil Hiley and published in 1993 (after Bohm's death in 1992). Bohm and Hiley wrote:

> Moreover if one is not aesthetically satisfied with this picture of a static electron in a stationary state, one can go to the stochastic model given in Chapter 9. In this model the particle will

> have a random motion round an average $p = \nabla S$, and the net probability density in this random motion comes out as $P = |\Psi(x)|^2$ [13, p. 43].

David Bohm's ideas were very important. One of the most relevant results of these ideas was to establish that it is possible to overcome the "impossibility proofs", such as von Neumann's theorem. An article clearly showing this fact is by Franco Selleri [37], where a simple model of a statistical ensemble of spinning spheres is enough to reproduce all the results (eigenfunctions, eigenvalues, probabilities) of the quantum theory of spin one-half. This result, which at the same time constitutes a criticism of both Bohr's complementarity and von Neumann's impossibility theorem, means that, at least from a methodological point of view, the idea of hidden variables is immensely valuable.

Karl Popper also protested against Bohr's view. He wrote: "But this doctrine is simply false: quantum theory is as objective as any theory can be" [32, p. 120], and also, "Quantum theory is exactly as objective as any other physical theory" [32, p. 121].

Gell-Mann also argued that:

> Niels Bohr brainwashed a whole generation of physicists into believing that the problem had been solved fifty years ago [22, p. 29].

Penrose, too, manifested his preference and arguments for a realistic interpretation of quantum mechanics and an objective description of reality. He wrote:

> Yet it must also be emphasized that, in my view, the standard theory is indeed quite unsatisfactory philosophically. Like Einstein and his hidden-variable followers, I believe strongly that it is the purpose of physics to provide an *objective* description of reality [31, p. 106].

Penrose's ideas, which he considered as going "into dangerously speculative territory", seem to be very close, broadly speaking, to those of the Einsteinian programme, except with respect to *indeterminacy*. Penrose argued that "[...] I do not regard *indeterminacy*, in the ordinary sense of that word, as being necessarily objectionable". I agree with this speculation because I believe that we live in a world of propensities, in which there are objective probabilities (concrete and real tendencies), not something existing in the mind of somebody (see [34,3]).

Penrose considers the co-existence of the deterministic evolution of the solutions of Schrödinger's equation and the non-deterministic jump that characterizes the quantum collapse "an absurd concoction". He argues in favour of attempts to write an improved theory and also in favour of a possible explanation of the collapse in gravitational terms. His research programme can be summarized as follows:

> I would not dispute that some changes in classical general relativity must necessarily result if a successful union with quantum physics is to be achieved, but I would argue strongly that these must be accompanied by equally profound changes in the structure of quantum mechanics itself. The elegance and profundity of general relativity is no less than that of quantum theory. The successful bringing of the two together will never be achieved, in my view, if one insists on sacrificing the elegance and profundity of either one in order to preserve intact that of the other. What must be sought instead is a grand union of the two – some theory with a depth, beauty and character of its own (and which will be no doubt recognized by the strength of these qualities when it is found) and which includes both general relativity and standard quantum theory as two particular limiting cases [31, p. 112].

I think that the above lucid quotation helps us to better analyse the state of the art of the discussion on the enormous existing difficulties. In the present stages of the development of general relativity and quantum theory, it can be said that they are extraordinarily good theories but that they are also incommensurable. If we believe that an eventual conciliation between them is possible, then radical improvements are necessary. It is unlikely that just one will have to be modified. The resulting theory (if possible, but I share Penrose's optimism) should contain both general relativity and standard quantum theory as particular limiting cases. This hope once more is in agreement with Einstein's statement:

> There could be no fairer destiny for any [...] theory than that it should point the way to a more comprehensive theory in which it lives on, as a limiting case.[7]

John Stuart Bell also expressed his dissatisfaction several times with respect to orthodox quantum mechanics as based on ambiguous concepts such as 'measurement'. Instead of an orthodox theory based on *observable* elements, we need a theory centred on '*beable'* ones. His programme is clearly outlined in this passage:

> In particular we will exclude the notion of 'observable' in favour of that of '*be*able'. The *be*ables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on 'observation'. Indeed observation and observers must be made out of beables [7, p. 174].

He follows this statement with:

> I use the term '*be*able' rather than some more committed term like 'being' to recall the essentially tentative nature of any physical theory. Such a theory is at best a *candidate* for the description of nature. Terms like 'being', 'beer', 'existent', etc., would seem to me lacking in humility. In fact 'beable' is short for 'may-beable' [7, p. 174].

The last book written by Bohm, in collaboration with Hiley, explicitly affirms the need for an ontological interpretation of a quantum theory. Bohm died suddenly in 1992 and the book was published in January 1993. In the preface Hiley wrote:

> It was quite clear from the outset that it was not going to be possible to return to the concepts of classical physics and we found it necessary to make some radical new proposals concerning the nature of reality in order to provide a coherent ontology.[8]

In this book Bohm's thought is considerably more wide-ranging than in his 1952 papers. His idea of *implicate order* is extensively explained in the last chapter, which is entitled "Quantum Theory and Implicate Order". But Bohm and Hiley affirm that the ontological approach presented in the book does not mean any return to classical concepts. They emphatically hold that the quantum potential concept, for example, essentially belongs to a quantum rather than to a classical context.

Concluding this section, I would like to emphasize once again that the 'dissolution of reality' brainwashing, which still dominates the scene of the scientific community, has found important opponents. Einstein himself was the most outstanding example, but he died in 1955. Concerning physical reality, Popper was an important realist and rationalist philosopher, even though his political views should be – and have been – severely criticized. Popper died in 1994. Bohm, who died in 1992, emphasized the necessity of an objective ontological theory without having to assume an outside observer. Bell, who died

[7] Einstein as cited in [33] (inscription to Chapter 1, p. 32).

[8] Hiley, preface to [13].

in 1990, had his 'beable' programme which is a realistic and rationalist one in the sense of the Einsteinian and Popperian conceptions. There are several research groups committed to a rationalist and realistic approach to physical inquiry. Selleri, who has published several books and articles and organized several conferences in which researchers belonging to all the tendencies (for or against realism) are invited, is an important example of a living physicist committed to realism and rationality. Penrose, with his vision of a future theory which must encompass both general relativity and quantum mechanics as limiting cases, in the spirit of a generalized correspondence principle, offers a concrete possibility. In the next section we will consider other aspects of the generalized 'beable' programme which can be summarized by the following claim: all physical theories are attempts at an objective description of nature. Quantum theory is no exception. In short, the 'beable' programme is the same as a search for a *quantum theory without observers*.

10. A Quantum Theory Without Observers

In the last section, I documented the views of some of the physicists who criticized the orthodox interpretation of quantum physics ascribing observers a central role in the theory. Bohm, Hiley, Selleri, Gell-Mann and Penrose, for example, argued in favour of different programmes that are committed to excluding 'the observer' from quantum physics. In other words, all these programmes require an ontological interpretation in which the observer does not play any essential role.

Following Bell's idea – abandoning the notion of 'observable' in favour of that of 'beable' – over the last two decades some authors have made an effort to understand the emergence *from* the quantum world (characterized by a peculiar entanglement of states) *to* the classical world with its separated and disentangled objects. This emergence can also be understood as a passage *from* the "coherent" world (ruled by the superposition principle and by interference of quantum amplitudes of probability) *to* the "decoherent" classical world which moulds our ordinary physical intuition. Among the authors working on theories of this kind are Griffiths, Omnès, Gell-Mann, Hartle and Zurek (see [23]).

The modern decoherence theories have the important objective of overcoming the Copenhagen wave-function collapse. In my opinion, these authors broadly accept the 'beable' programme, as shown by the following questions asked by them: "What is it in the laboratory that corresponds to a wave function, or to an angular momentum operator?" [23, p. 26], and later: "What are the 'beables' [...] of quantum theory – that is to say, the physical referents of the mathematical terms?" [23, p. 26]. They recognize that the connection of the mathematical structures of quantum theory with physical reality through the concept of measurement leads to immense difficulties and "ludicrous" consequences: "When quantum mechanics is applied to astrophysics and cosmology, the whole idea of using measurements to interpret its predictions seems ludicrous" [23, p. 26].

With respect to wave-function collapse, Haroche writes:

> [...] the proponents of the modern decoherence theories prefer to view this wavefunction collapse [Copenhagen interpretation] as a real physical process caused by the coupling of the measuring apparatus to its environment. For all practical purpose, of course, the orthodox and decoherence points of view are equivalent, because the decoherence time is infinitesimal for any measurement that ultimately involves a macroscopic apparatus [24, p. 41].

I think there exists a misunderstanding in this kind of evaluation. The question is not centred on a possible "practical equivalence" of the decoherence point of view and the wave-function collapse. If one adopts the decoherence point of view, it is obvious that an eventual and very special quantum relaxation must necessarily take place during a finite time as a physical process. The central question is that a consistent decoherence theory must be formulated independently of measurements. Only in this case the "ludicrous" aspects censored by Griffith and Omnès (more than 60 years after Einstein and Schrödinger) are removed.

The same author continues:

> Decoherence becomes more and more efficient as the size of a system increases. It protects with a vengeance the classical character of our macroscopic world [24, p. 42].

Griffiths and Omnès write:

> In the consistent-histories approach, the concept of measurement is not the basis for interpreting quantum theory. Instead, measurements can be analyzed, together with other quantum phenomena, in terms of physical processes. And there is no need to invoke mysterious long-range influences and similar ghostly effects that are sometimes claimed to be present in the quantum world [23, p. 26].

Griffiths and Omnès consider that the decoherence point of view constitutes a good way to progress. They argue that:

> [...] calculations carried out by one of us and by Gell-Mann and Hartle, indicate that, given a suitable consistent family, classical physics does indeed emerge from quantum theory [23, p. 31].

In spite of the rational aim of overcoming instantaneous action at a distance, ghostly effects and subjective interpretations like the attribution of physical effects to conscience, decoherence theories do not constitute – in my opinion – a completely satisfactory point of view. Decoherence is simply a reinterpretation of the Copenhagen orthodoxy, trying to overcome some of its absurdities. From this viewpoint, decoherence theories seem to offer no solid basis to unify and approximate quantum theory and classical general relativity in the sense proposed by Penrose.

We conclude this section by quoting two very interesting passages of Gerard 't Hooft [43].

> To this day, many researchers agree with Bohr's pragmatic attitude. The history books say that Bohr has proved Einstein wrong. But others, including myself, suspect that, in the long run, the Einsteinian view might return: that there is something missing in the Copenhagen interpretation. Einstein's original objections could be overturned, but problems still arise if one tries to formulate the quantum mechanics of the entire universe (where measurements can never be repeated), *and* if one tries to reconcile the laws of quantum mechanics with those of gravitation [43, p. 13].

> You may have already suspected that I still believe in the hidden variables hypothesis. Surely our world must be constructed in such an ingenuous way that some of the assumptions that Einstein, Bell and others found quite natural will turn out to be wrong. But how this will come about, I do not know. Anyway, for me the hidden variables hypothesis is still the best way to ease my conscience about quantum mechanics [43, p. 15].

All these circumstances show that the Einstein Programme lives, and that the mythology of a "loser" Einstein versus the "winner" Bohr must be replaced by a new and deeper understanding of Einstein's thought.

11. Do 'Crucial' Experiments Exist?

The obsession of the overwhelming majority of scientists with the search for crucial experiments is well known. These scientists search for experiments that can show, decisively, who is right and who is wrong, or which is the correct theory and which is (are) the false(s) one(s). Experimental control is a consolidated practice of science and the dialogue between theory and experiment plays a central role in the development of science. This dialogue gives rise to new possibilities, but very rarely leads to the last word on a subject (see [15,8], [28, p. 323], [29, pp. 44–46], [4]). A set of 'crucial experiments' is a confrontation among several webs of theories and this situation is completely different from the kind of confrontation imagined by some naïve realists.

The history of science shows several characteristic examples. The Fizeau and Foucault experiments in 1850 'decided' in favour of the wave character of light, but in the twentieth century Einstein 'invented' duality for light and de Broglie 'invented' duality for all 'particles'. Also, Michelson and Morley's experiments cannot be considered as crucial in discarding the existence of an ether.

The situation is even more confusing in the context of the confrontation *locality* versus *non-locality*. As we know, Einstein argued that "physics should represent a reality in space–time, free from any spooky action at a distance" [19, p. 158]. With a great acuteness and awareness of the EPR paradox (see Appendix B), Schrödinger wrote on the strange quantum entanglement:

> It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it [35].

Bell was able to find a formula – the famous Bell inequality (see Appendix C) – which gives rise to the possibility of a possible "crucial experiment" to settle the nature of this entanglement. In other words, Bell's inequality offered the possibility of an experimental confrontation of quantum mechanics (theory implying entanglement) with local theories or local models. Some extremely difficult experiments were carried out, but whether any of them can be considered as "crucial" cannot be considered as decided among scholars in the field. Hiley and Peat, for example, put the important questions:

> 1. Whether Bell's notion of locality is too restrictive; and 2. Whether in fact the experiments actually measure what they intend to measure [25, p. 14].

Although the overwhelming majority of physicists believe that the final word has already been said, we will finish this section by quoting one of the dissenting voices:

> In EPR experiments, the experimenters do what they are asked to do: find conditions in which Bell's inequalities were infringed! Nobody, it seems, puts any restraints on the methods they used, or asks them to publish full data, including the runs that do not quite work. The magicians know how to *produce* their illusions (albeit not quite perfectly – witness those "anomalies"), but why do they still not *understand* them? [42, p. 358].

12. Concluding Remarks

The debate on the foundations of quantum theory is inconclusive. My feeling is that the basic problems of modern physics are enormously complex, and are still largely unsolved. Regrettably, a great number of scientists thinks that the final word on the subject has been given through some supposedly 'crucial' experiments. The great difficulty of the problems involved suggests that this view is a naive one. The Einsteinian programme of an eventual unification has still several adherents.

We have arrived at the following general conclusions:

(1) A large proportion of the misunderstanding, myths, brainwashing and conceptual confusion on the quantum theory comes from a limited understanding of the science involved, which is unable to conceive the enormous complexity of this kind of activity.
(2) Physics (and, specifically, quantum mechanics) is a very complex science made up of several scientific research programmes. In physics there are rival (competitive) programmes as well as cooperative ones. They form a web of theories and methods, but this does not imply inconsistency or any accommodation to contradictions. In incorporating rival programmes, physicists always aim at consistency, i.e. eliminating any contradictions.
(3) In this way, the co-existence and fertility of programmes like "Everything is Field" with Programmes like "Everything is Particle" does not mean a contradiction. This co-existence shows only the enormous complexity of physical reality.
(4) This complexity cannot be interpreted as any kind of "cultural relativism".
(5) The myth of the "dissolution of reality" constitutes dangerous brainwashing which threatens the objectivity of science and hinders the elimination of contradictions.

We believe that Einstein's thought was headed firmly in this rational and realistic direction. When Einstein said that, "It is theory that decides what is observable", he wanted to emphasize this important aspect of the complexity of the dialogue theory/experience which was a central part of his thought.

Appendix A. Bohm's Theory

As we know, in general a complex function $\Psi = \mathrm{Re}(\Psi) + i\,\mathrm{Im}(\Psi)$, where $i = (-1)^{1/2}$ and $\mathrm{Re}(\Psi)$ and $\mathrm{Im}(\Psi)$ are real quantities, can be written in polar form as

$$\Psi = R \exp\left(\frac{2\pi i S}{h}\right) \tag{A.1}$$

Inserting (A.1) into Schrödinger's equation

$$\frac{ih}{2\pi}\frac{\partial\Psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\nabla^2\Psi + V\Psi \tag{A.2}$$

we obtain, after separating purely real terms and purely imaginary terms, the following two equations

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{h^2}{8\pi^2 m}\frac{\nabla^2 R}{R} = 0, \tag{A.3}$$

$$\frac{\partial R^2}{\partial t} + \nabla.\left(\frac{R^2 \nabla S}{m}\right) = 0, \tag{A.4}$$

where S is the classical action of the system, V is the classical potential acting on the particle of mass m, and ∇ is the gradient operator which in Cartesian coordinates is given by

$$\nabla = \frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z,$$

where $\hat{e}_x,\ \hat{e}_y,\ \hat{e}_z$ are the unit vectors respectively in the directions x, y and z, and ∇^2 is the Laplacian operator which is a scalar operator resulting from the scalar product $(\nabla.\nabla)$, which in Cartesian coordinates is given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

If we consider the Hamilton–Jacobi theory according to which

$$\frac{\partial S}{\partial t} = -E \tag{A.5}$$

and

$$\nabla S = p, \tag{A.6}$$

Equation (A.3) can be written in the form

$$E = \frac{p^2}{2m} + V - \frac{h^2}{8\pi^2 m}\frac{\nabla^2 R}{R}. \tag{A.7}$$

Without the last term, the above equation becomes the well-known classical energy of the system which is the sum of kinetic plus potential energies. In the context of the WKB approximation the term $-(h^2/8\pi^2 m)(\nabla^2 R/R)$ is neglected (see [13, p. 28]). Alternatively, we can say that the classical Hamilton–Jacobi theory is reproduced from quantum mechanics in the first order of $(h/2\pi)$ and not in the zero-th order (see [26, Chap. III, § 17]).

Bohm argues that the last term plays an essential role. This term is the quantum potential

$$Q = -\frac{h^2}{8\pi^2 m}\frac{\nabla^2 R}{R}. \tag{A.8}$$

The quantum Hamilton–Jacobi Eq. (A.3) then becomes

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0. \tag{A.9}$$

On the other hand, Eq. (A.4) is usually interpreted in analogy with the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla . j = 0$$

of hydrodynamics and of electromagnetism as one expressing the conservation of probability. The quantity R^2 (density of probability) is analogous to the density of charge (or mass) ρ and the quantity $R^2 \nabla S/m$ (density of current of probability) is analogous to the classical quantity j (superficial density of current). Let us go back to Eq. (A.9). If we accept the above description, we can allow that the equation of motion of the particle of mass m is given by

$$\frac{m\,\mathrm{d}v}{\mathrm{d}t} = -\nabla V - \nabla Q.$$

The above equation has the following meaning: the force acting on the particle of mass m has two components, each of a different nature. The first one is classical force $(-\nabla V)$ and the second is quantum force $(-\nabla Q)$. It is important to emphasize that according to Bohm and Hiley this description does not represent any return to the concepts of classical physics (see the quotation by Hiley in Section 9). The quantum potential Q constitutes a new concept which cannot be reduced to classical concepts.

In order to apply Bohm's ideas in quantum mechanics, we consider, for example, the s-states in a hydrogen atom. As we know, s-states have an orbital quantum number equal to zero and their corresponding wave-functions are real quantities. From Eq. (A.1) we can see that when this takes place then $S = 0$. Consequently, this circumstance leads to $p = 0$. In this case we obtain $E = V + Q$. According to this view, this result means that in the stationary states s electrons are at rest. This also means that the effects produced by the classical potential V on the electron are exactly balanced by the effects produced by the quantum potential Q on the same electron. Consequently the electron does not fall into the nucleus.

Appendix B. Orthodox Quantum Theory and the Problem of Measurement

Let us consider a pair of particles $\{\alpha, \beta\}$ described by the singlet state

$$|\Psi_S\rangle = (2)^{-1/2}\big\{|u^+\rangle|v^-\rangle - |u^-\rangle|v^+\rangle\big\}, \tag{B.1}$$

where $|u^\pm\rangle$ and $|v^\pm\rangle$ are eigenstates of the z-component of the spin operators $S_{\alpha z}$ and $S_{\beta z}$, respectively, where the subscript z denotes the z-component of the spin operator, the subscript α refers to the particle α and the subscript β refers to the particle β. The eigenvectors $|u^\pm\rangle$ and $|v^\pm\rangle$ are represented by their corresponding spinors.

We have

$$S_{\alpha z} = \frac{h}{4\pi}\sigma_\alpha; \qquad S_{\beta z} = \frac{h}{4\pi}\sigma_\beta, \tag{B.2}$$

where σ_α and σ_β are the 2×2 Pauli matrices respectively corresponding to the particles α and β.

Let us consider the square of the total spin of the system $\{\alpha, \beta\}$ whose corresponding operator is given by

$$S^2 = (S_\alpha + S_\beta)^2 \tag{B.3}$$

and the z-component operator of the total spin of the system $\{\alpha, \beta\}$

$$S_z = S_{\alpha z} + S_{\beta,z}. \tag{B.4}$$

Straightforward calculations lead to

$$S^2|\Psi_S\rangle = 0|\Psi_S\rangle, \tag{B.5}$$

$$S_z|\Psi S\rangle = 0|\Psi_S\rangle, \tag{B.6}$$

$$S_{\alpha z}|u^{\pm}\rangle = \pm\frac{h}{4\pi}|u^{\pm}\rangle, \tag{B.7}$$

$$S_{\beta z}|v^{\pm}\rangle = \pm\frac{h}{4\pi}|v^{\pm}\rangle. \tag{B.8}$$

From (B.5) and (B.6) it follows that:

$$\langle\Psi_S|S^2|\Psi_S\rangle = 0, \tag{B.9}$$

$$\langle\Psi_S|S_z|\Psi_S\rangle = 0, \tag{B.10}$$

i.e. in the singlet state the average values of the operators S^2 and S_z are both equal to zero.

The system made up of particles $\{\alpha, \beta\}$, which were initially together, decays and as a consequence particle α goes to a direction diametrically opposite to the direction in which particle β goes. In this new situation the particles are separated in space, but the system $\{\alpha, \beta\}$ continues to be described by the singlet state $|\Psi_S\rangle$.

We now suppose that a classical apparatus A performs a measurement on particle α *before* the classical apparatus B performs a measurement on particle β.

According to the usual orthodox quantum mechanics (associated with Bohr's theory of measurements) this measurement leads the initially entangled singlet state $|\Psi_S\rangle$ to one of two possible disentangled states, respectively, $|u^+\rangle|v^-\rangle$ or $|u^-\rangle|v^+\rangle$.

In other words, after the above measurement one of the two possibilities happens, whereas the other 'collapses'. Following this description, we have two possibilities for the average value of the operator S^2 in the new state. They are:

First possibility:

$$\langle u^+v^-|S^2|u^+v^-\rangle = \frac{h^2}{4\pi^2}, \tag{B.11}$$

Second possibility:

$$\langle u^-v^+|S^2|u^-v^+\rangle = \frac{h^2}{4\pi^2}. \tag{B.12}$$

If we compare (B.9) with (B.11) or, alternatively, (B.9) with (B.12), we conclude immediately that a measurement made by classical apparatus A on particle α is enough to cause a change in physical reality in the system $\{\alpha, \beta\}$. This measurement makes the value of the operator S^2 of system $\{\alpha, \beta\}$ jump *from zero to* $h^2/4\pi^2$.

Now we perform a second measurement by classical apparatus B on particle β *after* the measurement performed by classical apparatus A on particle α.

This also leads to two possibilities. They are:

First possibility:

$$S_{\beta z}|u^+v^-\rangle = -\frac{h}{4\pi}|u^+v^-\rangle. \tag{B.13}$$

Second possibility:

$$S_{\beta z}|u^-v^+\rangle = +\frac{h}{4\pi}|u^-v^+\rangle. \tag{B.14}$$

We note that this second measurement is irrelevant because in the first case above, $-h/4\pi$ is the eigenvalue of the operator $S_{\beta z}$ belonging to the eigenvector $|v^-\rangle$ and this eigenvalue was implemented by the first measurement made by the classical apparatus A on particle α, which led to Eq. (B.11). The same can be said with respect to the second possibility. In this case, $+h/4\pi$ is the eigenvalue of the operator $S_{\beta z}$ belonging to the eigenvector $|v^+\rangle$ and this eigenvalue was implemented by the first measurement made by the classical apparatus A on particle α, leading to Eq. (B.12). In short, in both cases the second measurement is irrelevant and does not change the physical reality of the system $\{\alpha, \beta\}$. This change of physical reality is effected by the *first* measurement.

Let us suppose that the first measurement is made at time t_A and that the second measurement is made at time t_B, where $t_B > t_A$. The quantity $\delta t = (t_B - t_A)$ may be considered infinitesimal; thus, no matter how small time δt is after the first measurement, any second measurement cannot change the physical reality. We remember that, according to the Copenhagen School's interpretation, the physical reality is contained in the state vector. Following this description (originally by Bohr) the collapse of the wave function is conceived as a form of instantaneous action at a distance.

In 1935 Schrödinger analysed a similar situation and wrote (ironically):

> Attention has recently (A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47** (1935), 777) been called to the obvious but very disconcerting fact that even though we restrict the disentangling measurement to *one* system, the representative obtained for the *other* system is by no means independent of the particular choice of observations which we select for that purpose and which by the way are *entirely* arbitrary. It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it.[9]

Finally, we stress that the famous EPR paper of 1935 was centred on the concept of completeness: Einstein and his collaborators tried to prove that quantum mechanics, as formulated in 1927, was an incomplete theory. Bell's seminal work of 1964, on the other hand, concentrated more on the concept of locality. He was able to offer a criterion for deciding whether quantum mechanics works when correlated particles happen to be separated by arbitrarily large distances. In this Appendix we have shown that if quantum mechanics and Bohr's theory of measurement are both correct, then instantaneous action at a distance follows as an unavoidable consequence.

[9] Schrödinger [35] *apud*, epigraph to Selleri [39].

Appendix C. Bell's Inequality

There are very clear demonstrations of Bell's inequality. We restrict ourselves to some comments. We recommend here to the reader the review article by Selleri [39] which includes several examples of this important theorem.

Here, we consider the following situation: we study a large number N of decays of type $\varepsilon \to \alpha + \beta$ forming a given ensemble. Observer $\hat{O}_\alpha$ performs a measurement on particle α and obtains a given value of the dichotomic observable $A(a)$; in an analogous way, observer $\hat{O}_\beta$ performs a measurement on particle β and obtains a given value of the dichotomic observable $B(b)$. The distance between particles α and β is considered to be arbitrarily large.

For the first decay we have the result (A_1, B_1), for the second decay the result (A_2, B_2), for the third the result (A_3, B_3), etc. We can define the correlation function,

$$P(a, b) = (N)^{-1} \sum_i A_i B_i, \tag{C.1}$$

where the sum is considered from $i = 1$ to $i = N$. The dichotomic observables assume the values $A_i = \pm 1$ and $B_i = \pm 1$. Consequently, $A_i B_i = \pm 1$. This implies, of course, that

$$-1 \leq P(a, b) \leq +1. \tag{C.2}$$

With respect to observable A we can take the arguments a and a' which are assumed to be experimental parameters, fixed in the structure of the apparatus in any given experiment, and the same holds for observable B and its arguments b and b'.

We can form correlation functions involving the arguments

$$P(a, b); \quad P(a, b'); \quad P(a', b); \quad P(a', b').$$

One specific combination of the above four correlation functions is enough to express the independence of the physical reality of sub-systems separated in space by arbitrarily large distances. This occurs for quantity Δ given by

$$\Delta = \left|P(a, b) - P(a, b')\right| + \left|P(a', b) + P(a', b')\right| \leq 2 \tag{C.3}$$

which is Bell's inequality. In short, we can say that if quantity Δ has a value < 2, then the locality is preserved and systems that are separated in space are physically independent. Otherwise, if $\Delta > 2$, Bell's inequality is violated and the systems separated in space are not independent.

We apply now this ideas to the singlet state $|\Psi_S\rangle$. In this case the observable A is $(\sigma_\alpha . a)$ where σ_α are the 2×2 Pauli matrices and the point denotes a scalar product. We have:

$$\sigma_\alpha = \sigma_{\alpha x}\, \hat{e}_x + \sigma_{\alpha y}\, \hat{e}_y + \sigma_{\alpha z}\, \hat{e}_z,$$
$$a = a_x\, 1\, \hat{e}_x + a_y\, 1\, \hat{e}_y + a_z\, 1\, \hat{e}_z,$$

where 1 is the 2×2 unit matrix. In the same way, the observable B is $(\sigma_\beta . b)$ and,

$$\sigma_\beta = \sigma_{\beta x}\, \hat{e}_x + \sigma_{\beta y}\, \hat{e}_y + \sigma_{\beta z}\, \hat{e}_z,$$
$$a = b_x\, 1\, \hat{e}_x + b_y\, 1\, \hat{e}_y + b_z\, 1\, \hat{e}_z.$$

Our correlation function here is the average value of the operator

$$\{(\sigma_\alpha . a) \otimes (\sigma_\beta . b)\}$$

calculated in the singlet state $|\Psi_S\rangle = (2)^{-1/2}\{|u^+\rangle|v_-\rangle - |u^-\rangle|v^+\rangle\}$. We remember that the observable $(\sigma_\alpha . a)$ acts only on the spinors $|u^\pm\rangle$ while the observable $(\sigma_\beta . b)$ acts only on the spinors $|v^\pm\rangle$. So we have:

$$P(a, b) = \langle\Psi_S|(\sigma_\alpha . a) \otimes (\sigma_\beta . b)\}|\Psi_S\rangle = -(a_x b_x + a_y b_y + a_z b_z)$$

$$\therefore \langle\Psi_S|(\sigma_\alpha . a) \otimes (\sigma_\beta . b)\}|\Psi_S\rangle = -a . b = -\cos(a, b).$$

In general this result violates Bell's inequality. The maximum violation takes place for angles $(a, b) = \pi/4$; $(a, b') = 3\pi/4$; $(a', b) = \pi/4$; $(a', b') = \pi/4$. For these angles we have $\cos \pi/4 = \sqrt{2}/2$; $\cos 3\pi/4 = -\sqrt{2}/2$.

The corresponding quantity Δ will be

$$\Delta = \left|P(a, b) - P(a, b')\right| + \left|P(a', b) + P(a', b')\right|$$

$$= \left|\frac{\sqrt{2}}{2} - \frac{-\sqrt{2}}{2}\right| + \left|\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right| = \sqrt{2} + \sqrt{2} = 2\sqrt{2}.$$

The singlet state is entangled, i.e. the physical reality of particles α, β which are separated in space at arbitrarily large distances, are not independent. In other words, the singlet state leads to a violation of Bell's inequality.

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