Physics Before and After Einstein
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Chapter 8

Einstein and Quantum Theory

Seiya Abiko
Seirei Christopher College, 3453 Mikatahara-Town, Hamamatsu-City, 433-8558, Japan

1. Prelude: Einstein's Encounter with Planck's Theory

Albert Einstein wrote, in 1946, when he was 67, his autobiography, "Autobiographical Notes" (abbreviated as "Notes" below), which was published three years later in a book entitled *Albert Einstein: Philosopher–Scientist*, edited by Paul Arthur Schilpp [21]. According to Schilpp, initially Einstein was not willing to write it but, after much persuasion by Schilpp, Einstein changed his mind, because he realized that it was his obligation to mankind to inform them how and why he reached his conclusions and interpretations in each of his works [56].

At the start of "Notes", he states that he wrote it as his own 'obituary' and, therefore, one may believe that he tried to write it as accurately as possible. Moreover, for such a versatile writer as Einstein, we have every reason to believe that he was as lively at that time as he had been in his prime; in fact, he was to live for another, active, nine years.

In "Notes", Einstein indicates his two points of view according to which it is possible to assess physical theories in general. They are 'external confirmation' (the theory must not contradict empirical facts) and 'internal perfection' (the 'naturalness' or 'logical simplicity' of its premises). These two seem to constitute for him the invariable prerequisites of a physical theory. From both viewpoints, he criticizes mechanics as the foundation of physics. Then, he describes the two revolutionary events in physics: the introduction of the field concept by Faraday–Maxwell's electromagnetic theory, and Max Planck's investigation into thermal radiation (1900). This chapter starts from this second revolutionary event.

> In his youth, Planck studied Rudolf Clausius' *Mechanical Theory of Heat* (*Mechanische Wärmelehre*) in detail, and prepared his doctoral dissertation on the second law of thermodynamics. He continued his studies on thermodynamics and applied their results successfully to physical chemistry. On this basis, in 1888 he was appointed successor of Gustav Kirchhoff, as assistant professor at the University of Berlin and director of the Institute for Theoretical Physics [42].

In the vicinity of the university, there was the newly founded Physical-Technological Imperial Institute (Physikalische-Technische Reichsanstalt), where precise measurements of thermal-radiation spectra, needed for the technological purpose of high-temperature measuring, were pursued. To combine the thermal-radiation spectra with the temperature of an emitting body, a precise radiation distribution formula for black-body radiation[1] was necessary. After a number of trials, Planck finally found out the radiation formula (which was later named after him), which exactly reproduced the experimental data.

His next step was to derive this radiation formula from the three fundamental theories of classical physics: i.e. mechanics, electromagnetism, and thermodynamics. After considerable efforts, he finally adopted Ludwig Boltzmann's statistical method. Boltzmann, in his gas theory, had assumed that the amount of kinetic energy each molecule could take is only an integral multiple of a minimum value ε. After the statistical calculation utilizing the probability theory, Boltzmann set $\varepsilon = 0$ and thus succeeded in deducing the equilibrium energy distribution formula of gas molecules he was aiming at.

Planck had already derived, utilizing electromagnetic theory and thermodynamics, the relationship between black-body radiation energy-density $\rho(\nu, T)$,[2] and the average energy value of the radiation-emitting elements, named as 'resonators' (i.e. minute antennas modelling atoms and molecules) with proper frequency ν at equilibrium with radiation. In order to obtain the latter average energy value of 'resonators', he applied Boltzmann's method to calculate the number of ways in which the total amount of energy is divided among 'resonators'. In a similar manner to Boltzmann's method, Planck deduced his radiation formula successfully, except for one crucial difference. In order to obtain his radiation formula, Planck had to set $\varepsilon = h\nu$ instead of $\varepsilon = 0$, where h is a proportionality constant which was later named Planck's constant.

On this matter, Einstein stated in "Notes" [19]:

> Planck got his radiation-formula if he chose his energy-elements ε of the magnitude $\varepsilon = h\nu$. The decisive element in doing this lies in the fact that the result depends on the taking for ε a definite finite value, i.e. that one does not go to the limit $\varepsilon = 0$. This form of reasoning does not make obvious the fact that it contradicts the mechanical and electrodynamic basis, upon which the derivation otherwise depends. Actually, however, the derivation presupposes implicitly that energy can be absorbed and emitted by the individual resonator only in 'quanta' of magnitude $h\nu$, i.e. that the energy of a mechanical structure capable of oscillation as well as the energy of radiation can be transferred only in such quanta – in contradiction to the laws of mechanics and electrodynamics. [...] All of this was quite clear to me shortly after the appearance of Planck's fundamental work; so that, without having a substitute for classical mechanics, I could nevertheless see to what kind of consequences this law of temperature-radiation[3] leads for the photo-electric effect[4] and for other related phenomena of transformation of radiation-energy, as well as for the specific heat of (especially) solid bodies. All my attempts, however, to adapt the theoretical foundation of physics to this new type of knowledge failed completely. It was as if the ground had been pulled out from under one, with no firm foundation to be seen anywhere, upon which one could have built.

If we believe that this statement is accurate, Einstein was shocked by Planck's theory "shortly after" its appearance in 1900, and delved deeply into the foundations of physics even before he published his first paper in 1901. The excerpt above from "Notes" tells us that his concern with black-body theory began at the same time as his scientific career.

In conflict with the above, Thomas S. Kuhn stated in his book *Blackbody* of 1978 [47]:

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[1] 'Black-body radiation' is a kind of thermal radiation which is at equilibrium with the emitting body kept at a constant temperature.

[2] The radiation energy-density coincides with the radiation distribution formula apart from a numerical factor.

[3] 'Temperature-radiation' is another name for 'thermal radiation'.

[4] Photo-electric effect' is a phenomenon of the emission of electrons (or cathode rays) from a body irradiated by (usually ultraviolet) light, discovered by Heinrich Hertz in 1887.

> Einstein's paper [i.e. the 1903 paper of the statistical trio from 1902 to 1904], written *before he showed any signs of concern with black-body theory*, bridged these gaps. [Italics added]

The "gaps" Kuhn referred to are Planck's uncertainty about the probability of states and his lack of direct statistical expressions for entropy[5] and temperature. In my view, however, it was in order to bridge "these gaps" in Planck's theory that Einstein constructed his theory of the statistical trio papers from 1902 to 1904, as will be explained below.

Usually, this kind of 'Autobiography' or 'Recollections' written in later years is regarded as unreliable due to the time that has elapsed between the events and writing about them. But, as will be shown below, we can confirm the accuracy of his description by comparing the text with numerous other documents Einstein left, e.g. his letters, papers and lectures.

In fact, the love letters he wrote in 1901 to his fiancée Mileva Marič contain the following remarkable statements [57]:

> I've begun to have reservations of a fundamental nature about Max Planck's studies of radiation, so much so that I'm reading his paper with mixed feeling. [4 April 1901]
>
> Planck assumes that a specific kind of resonator (fixed period and damping) causes the conversion of the radiation energy, an assumption that I have difficulty accepting. [10 April 1901]

Here, Einstein astutely discerned the problem in Planck's reasoning based on the classical theories of resonators.

We can also find in his love letters descriptions corresponding to the passage in "Notes" quoted above, "for the photo-electric effect and for other related phenomena of transformation of radiation-energy, as well as for the specific heat of (especially) solid bodies":

> It seems to me that it is not out of question that the latent kinetic energy of heat in solids and fluids can be thought of as the electric energy of resonators. If this is the case, then the specific heat and the absorption spectrum of solids would have to be related. [23 March 1901] [58].
>
> I was recently struck by the idea that when light is generated, a direct transformation from energy of motion into light might occur because of the parallel: the kinetic energy of the molecules – the absolute temperature – spectrum (spatial radiation energy in the state of equilibrium). [30 April 1901] [59].
>
> I have just read a wonderful paper by Lenard on the generation of cathode rays by ultraviolet light. Under the influence of this beautiful piece I am filled with such happiness and joy. [28 May 1901] [60].

These quotations from Einstein's letters prove to us that the quoted description from his "Notes" is accurate.

2. Did Planck Introduce a Physical Energy Quantum?

With regard to Einstein's comment quoted in the preceding section, "[Planck's] form of reasoning does not make obvious the fact that it contradicts the mechanical and electrodynamic basis", there arises the question: didn't Planck introduce the 'quantum postu-

[5] 'Entropy' is a physical quantity describing the state of a system (a substance or a collection of substances) introduced by Rudolf Clausius in 1865. He introduced it in order to express the second law of thermodynamics mathematically by describing the irreversibility of an isolated system as the increase of entropy.

late' that showed explicit contradiction with the mechanical and electrodynamic basis? This was also the point raised by Kuhn in *Blackbody*, quoted earlier. Kuhn's answer was negative [49].

In order to assess Kuhn's judgment, let us consult the portion of Planck's 1900 paper in which he is usually regarded as having introduced his 'quantum postulate' [54]:

> This constant [i.e. Planck's constant h] multiplied by the common frequency ν of the resonators gives us the energy element ε in ergs, and, dividing E by ε, we get the number P of energy elements which must be divided over the N resonators. *If the ratio thus calculated is not an integer, we take for P an integer in the neighbourhood.* [my italics]

What matters to us here is the italicized sentence, where Planck clearly indicates that the total energy E of resonators is not necessarily an integral multiple of the energy element ε. Therefore, the so-called 'quantum postulate' Planck is said to have introduced should be regarded merely as:

(1) the introduction of a natural constant h,
(2) the division of the energy scale into intervals composed of 'energy elements' $\varepsilon = h\nu$.

The first half (the introduction of h) was an inescapable consequence of Wien's displacement law.[6] In fact, the value of h had already been introduced by Planck into Wien's radiation formula (under the notation 'b') in his 1899 paper [53], that is, *before* the construction of his own radiation formula. Also, it was only in 1906, i.e. after the introduction of the term 'light-quantum' (Lichtquantum) by Einstein, that Planck designated h as the 'quantum of action' (Wirkungsquantum) [55]. On the other hand, the second half (the division into intervals of ε) was merely a mathematical device in order to deduce a finite value for the number of divisions. Therefore, the "energy element" Planck referred to had no physical basis.

That Einstein also shared this interpretation of Planck's text can be ascertained by consulting Einstein's 1906 review [32] of Planck's book, *Lectures on Thermal Radiation* (*Vorlesung über die Theorie der Wärmestralung*, Leipzig, 1906). Here, Einstein used the term 'elementary regions' (Elementargebiete) instead of 'energy elements' (Energieelemente), the term used by Planck, which confirms that Einstein denied Planck's introduction of the discreteness of energy or energy quantization.

3. Einstein's Research Programme: Construction of Statistical Thermodynamics

As I pointed out in Chapter 4, Section 7, there was an error in "Notes" in the first and the second editions of *Albert Einstein: Philosopher-Scientist*. The relevant portion after the correction reads [22]:

> Reflections of this type made it clear to me as long ago as shortly after 1900, i.e., shortly after Planck's trailblazing work, that neither mechanics nor *electrodynamics* (except in limiting cases) claim exact validity. By and by I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts. [Italics added]

[6] Wien's displacement law states that ν and T always appear in radiation formulas in the form ν/T. This law was discovered and derived by Wilhelm Wien in 1893.

The above quotation tells us that, "shortly after Planck's trailblazing work", he realized "that neither mechanics nor electrodynamics can claim exact validity". Thus, of the three fundamental theories of classical physics (i.e. mechanics, electromagnetism, and thermodynamics), Einstein regarded thermodynamics as "the only physical theory of universal content which, within the framework of the applicability of its basic concepts, will never be overthrown" [18]. Therefore, it is certain that the first core ingredient of Einstein's research programme was 'thermodynamics'.

But, as is well known, the applicability of the basic concepts of thermodynamics is much restricted. Therefore, he should have felt it necessary to broaden its applicability. His love letters quoted in Section 1, which show that he was studying closely the microscopic mechanisms of energy transformation, could be read as indicating the direction of his efforts to broaden the applicability.

After the quoted part from his "Notes" in Section 1, the following lines read [20]:

> My own interest in these years was less concerned with the detailed consequences of Planck's results, [...] My major question was: What general conclusions can be drawn from the radiation-formula concerning the structure of radiation and even more generally concerning the electromagnetic foundation of physics?

This paragraph can be interpreted as the specification of his research programme in his early years, i.e. his aim was to draw conclusions concerning "the structure of radiation" and "the electromagnetic foundation of physics" from Planck's radiation formula.

Several other portions of his love letters (e.g. those dated 13 September 1900 [61], 3 October 1900 [63] and 15 April 1901 [62]) reveal the actual development of his research programme. According to them, he was much attracted to Boltzmann's kinetic theory of gases, the ionic theory of physical chemistry in conflict with Maxwell's electromagnetic theory, and the influence of intermolecular forces on the dissipative processes. As a matter of fact, these themes correspond to the five papers he wrote before 1905. As I pointed out elsewhere [2], in those early years Einstein was concerned with the connection between macroscopic and microscopic aspects of matter. In fact, his first and second papers try to apply the expression of intermolecular forces, which he conjectured modelling the law of universal gravitation, to the macroscopic phenomena of capillarity and of the contact electric-potential difference.

Following the above two papers on concrete physical phenomena, between 1902 and 1904 he published three papers of a more general scope on statistical physics (referred to below as 'the statistical trio') [14]. Being unsatisfied with Planck's theory, Einstein's motive for composing the statistical trio must have been somehow to construct a general thermodynamics, on which the theories, not only of fluids and solids, but also of thermal radiation could be based.

Since Planck's theory was modelled on Boltzmann's theory, Einstein's aim in the first paper of the statistical trio was to attempt to close the gap in Boltzmann's kinetic theory of heat. The gap Einstein referred to is the lack of a derivation of the laws of thermal equilibrium and the second law of thermodynamics using only the equations of mechanics and the probability calculus. More precisely, he constructed a statistical theory which treats an ensemble of systems enclosed by an environment kept at constant temperature T, i.e. the case later called the 'canonical ensemble'.

In this first paper of the statistical trio, he also derived the mathematical expression for entropy from the standpoint of mechanics, upon which he made an important comment [14, p. 47]:

> This expression for the entropy is remarkable, because it depends only on E and T, while the special form of E as a sum of potential energy and kinetic energy no longer appears. This fact suggests that the result we obtained is more general than the mechanical representation here adopted.

In accordance with the above, the second statistical paper, published in 1903, begins by asking the question [14, p. 48]:

> Whether the kinetic theory is really necessary for the derivation of the foundations of the theory of heat obtained in the 1902 paper, or assumptions of a more general nature may suffice?

Then he showed that, in fact, the latter is the case. He derived, without resorting to the kinetic theory, the foundations of the theory of heat with regard to the most general system expressed, not by coordinates and momenta, but by 'state-variables' (*Zustandsvariabeln*) $p_1, \ldots, p_n$.

The generality of the system treated is restricted only by two conditions: first, the existence of causal equations for the state-variables:

$$\frac{dp_i}{dt} = \phi_i(p_1 \cdots p_n) \quad (i = 1, \ldots, n) \tag{1}$$

and, second, the existence of the energy equation for these causal equations

$$E(p_1 \cdots p_n) = const. \tag{2}$$

Accordingly, his theory of statistical physics underwent a transformation from 'statistical mechanics' into 'statistical thermodynamics' at this point.

The last paper of the trio, published in 1904, was written as an addendum to the second. Here Einstein made the following important remark [14, p. 68]:

> I derive the expression of the entropy of a system, which is completely analogous to that found by Boltzmann[7] for ideal gases and assumed by Planck in his theory of radiation.

Einstein derived an expression for the entropy S of the general thermodynamical system treated in the second paper, as follows (using the modern notation of Boltzmann's constant k instead of 2κ in Einstein's paper):

$$S = k \log\big[\omega(E)\big], \tag{3}$$

where

$$\omega(E)\delta E = \int_E^{E+\delta E} dp_1 \cdots dp_n. \tag{4}$$

Though he did not give this any name, we will call $\omega(E)$ the 'state-density' (as a function of energy).

As the system under consideration is enclosed by an environment kept at constant temperature T, the system can exchange energy with the environment. As a result, the

[7] Boltzmann expressed the entropy of a state of the system under consideration by a logarithm of the realization-probability of the state. This expression, which Planck utilized in the derivation of his radiation formula, was later named 'Boltzmann's principle' by Einstein.

energy E of the system fluctuates around its average value $\overline{E}$.[8] If we put $E = \overline{E} + \varepsilon$, then $\overline{\varepsilon^2} = \overline{E^2} - \overline{E}\,\overline{E}$ represents a measure of the thermal stability of the system, in the sense that the smaller its value, the greater the stability. With regard to this quantity, he derived, utilizing Eqs (3) and (4), the expression [14, p. 75]:

$$kT^2 \frac{d\overline{E}}{dT} = \overline{E^2} - \overline{E}\,\overline{E}. \quad (5)$$

Thus, he concluded that the constant k determines "the thermal stability of the system".

Next, he looked for a system for which it was possible to determine $\overline{\varepsilon^2}$, and stated [14, p. 76]:

> In fact, there is only a single kind of physical system for which we can surmise from experience that it possesses energy fluctuation: this is empty space filled with temperature radiation.

Thus, he finally applied his theory to black-body radiation. Applying Stefan–Boltzmann's law[9] and the value of k obtained from the kinetic theory of gases to Eq. (5), he could successfully obtain a version of Wien's displacement law $\lambda_m \propto 1/T$, i.e. the correct dependence of the wavelength λ_m of the maximum radiation energy on the temperature T. What is more, the proportionality coefficient deduced was in order of magnitude agreement with the observed value.

We see here that Einstein went one step further to his final aim. What enabled him to do so was his departure from 'statistical mechanics' to the more general 'statistical thermodynamics'. Statistical thermodynamics, and the theory of fluctuation thus constructed, underlay his doctoral dissertation, the theory of Brownian movement, the theory of light quantum (so far 1905), the first quantum theory of matter (1906), and the theory of specific heat (1907). Furthermore, as I discussed in Chapter 4, there was a close relationship between his special theory of relativity and the statistical thermodynamics.

4. Investigation into the Constitution of Radiation: The Light-Quantum Theory

In the last quarter of the nineteenth century, the phenomenon of Brownian movement was becoming widely known. While a minority of scientists still attributed its cause to such effects as electrical effects, osmosis, or surface tension, most seemed to think that it must be connected with thermal molecular motions. However, on account of the immense difference in size between small particles and molecules, those who were familiar with the kinetic theory of gases were suspicious of this view. Moreover, there was still no quantitative theory to be tested against experiments [8].

Not only did Einstein's theory of Brownian movement [25] provide a breakthrough in the understanding of this phenomena, it also did, to borrow Max Born's wording, "more than any other work to convince physicists of the reality of atoms and molecules, of the kinetic theory of heat, and of the fundamental part of probability in the natural laws" [7].

[8] The horizontal bar designates the average over the ensemble of systems, which is assumed to coincide with the temporal average of one of the systems.

[9] Stefan–Boltzmann's law states that the total radiation energy emitted per second from a black-body kept at temperature T takes a finite value proportional to T^4. This law was discovered by Joseph Stefan in 1879 and derived by Boltzmann in 1884.

"Notes" proceeds, after a description of the statistical trio and their successful application to Brownian movement, to a thought-experiment[10] on the Brownian movement of a small mirror suspended in a cavity radiation.[11] This thought-experiment required the assumption that "radiation energy consists of indivisible point-like localized quanta of energy $h\nu$", as follows [23]:

> Upon this recognition [i.e. the success of the theory of Brownian movement], a relatively direct method can be based which permits us to learn something concerning the constitution of radiation from Planck's formula. One may conclude in fact, in a space filled with radiation, a freely moving, quasi monochromatically reflecting [two-sided] mirror would have to go through a kind of Brownian movement, the average kinetic energy of which equals $kT/2$.[12] If radiation were not subject to local fluctuations, the mirror would gradually come to rest, because, due to its motion, it reflects more radiation on its front than on its reverse side.[13] However, the mirror must experience certain random fluctuations of the pressure exerted upon it due to the fact that the wave-packets, constituting the radiation, interfere with one another. These can be computed from Maxwell's theory. This calculation, then, shows that these pressure variations are by no means sufficient to impart to the mirror the average kinetic energy $kT/2$. In order to get this result, one has to assume rather that there exists a second type of pressure variations, which cannot be derived from Maxwell's theory, which corresponds to the assumption that radiation energy consists of indivisible point-like localized quanta of energy $h\nu$, which are reflected undivided.

The above description continues with the passage cited at the start of Section 3 (i.e. the passage in which the correction was made) and then ends with remarks on special and general relativity. Therefore, we can see that the analysis of the Brownian movement of a suspended mirror was made well before 1905 ("shortly after 1900"), i.e. before his famous light-quantum theory was constructed. Therefore, let us take up this thought-experiment of a suspended mirror, before entering into the light-quantum theory.

The thread of his thought seems to have been the following. After reading Planck's 1900 paper, Einstein thought that, in order to derive Planck's formula, the resonator energy should be restricted to discrete values. But this means, from the viewpoint of energy conservation, that the energy of thermal radiation itself should also take only a discrete set of values. Therefore, he needed a more direct way to corroborate this inference. This requirement subsequently led him to turn to the thought-experiment of Brownian movement of a suspended small mirror, which was what he meant by "a relatively direct method [...] to learn something concerning the constitution of radiation from Planck's formula". As we will show shortly, it was by this thought-experiment that he introduced the so-called wave-particle duality for the first time.

The concrete procedure of the calculation is given in papers written later in 1909 and 1910. According to the equipartition law of statistical thermodynamics, in the environment kept at constant temperature T, the mean-square value $\overline{v^2}$ of the mirror velocity

[10] A thought-experiment is an imaginative experiment considered in order to make inferences theoretically.

[11] Black-body radiation can be realized experimentally by the radiation confined within a cavity surrounded by walls kept at constant temperature. This type of radiation is called 'cavity radiation'.

[12] According to the equipartition law of statistical thermodynamics, in a system kept at constant temperature T, the average kinetic energy of a body per a degree of freedom (i.e. one of possible modes) of its motion should be $kT/2$.

[13] Maxwell's electromagnetic theory states that there is radiation pressure in the direction normal to the electromagnetic waves, and numerically equal to the radiation energy in unit volume.

v in the Brownian movement must take the value (using k instead of R/N in Einstein's paper):

$$\overline{v^2} = \frac{k}{m}T. \tag{6}$$

In the 1909 paper [26], Einstein introduced the increment Δ of the velocity v of the two-sided small mirror during a short time-interval τ. This increment of the velocity is brought about by the random fluctuation of the pressure of radiation exerted on the mirror. The condition that the value $\overline{v^2}$ should remain unchanged during τ leads to

$$\overline{\Delta^2} = \frac{2P\tau}{m}\overline{v^2}, \tag{7}$$

where m is the mass of the mirror, and P is the constant of radiation friction occurring due to the excess reflection on the front than the reverse side of the mirror (because of its motion). An expression of P is given, without its derivation in this paper, in terms of radiation energy-density ρ (as a function of ν and T per unit volume and per unit frequency range).

It is in the 1910 paper [40] that the derivation of the expression for P in terms of ρ was shown, where Maxwell's equation as well as the results of the special theory of relativity of 1905 were utilized. As stated in Chapter 4 of this book, it might be in order to fulfil this urgent requirement that the special theory of relativity was constructed at that time. This expression for P was then utilized, in combination with the calculated results for $\overline{\Delta^2}$ from Maxwell's equation, to yield the expression for ρ, the result of which was Rayleigh–Jeans' radiation formula:

$$\rho = \frac{8\pi\nu^2}{c^3}kT. \tag{8}$$

Returning to the 1909 paper, Einstein examined the consequence of the substitution, as ρ in the expression of P, of Planck's radiation formula:

$$\rho = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{(h\nu)/(kT)} - 1}. \tag{9}$$

The resultant $\overline{\Delta^2}$ in the frequency interval $\nu \sim \nu + d\nu$ with surface area f of the mirror was:

$$\frac{\overline{\Delta^2}}{\tau} = \frac{1}{c}\left[h\nu\rho + \frac{c^3}{8\pi}\frac{\rho^2}{\nu^2}\right]f\,d\nu. \tag{10}$$

In the above, the second term on the right-hand side is the same form as that derived in the 1910 paper utilizing Maxwell's theory. The first term on the right-hand side, however, indicates the same fluctuation as that exhibited in a collection of gas molecules each having energy $h\nu$. In other words, *the first term exhibits particle-like fluctuation, while the second term exhibits wave-like fluctuation.*

Thus, wave-particle duality, the most peculiar feature of quantum theory, was first introduced by Einstein in this 1909 paper [43]. If one inserts $\rho V\,d\nu$ as $\overline{E}$ into Eq. (5), with ρ substituted by Planck's formula Eq. (9), the same type of result as in Eq. (10) is obtained for the energy fluctuation of the radiation component ν. According to "Notes", all these considerations were carried out well before 1905.

Encouraged by the success of these investigations into radiation, Einstein proceeded to the full consideration of the consequences of Planck's formula, the result of which was his famous light-quantum theory of 1905 [24]. In this paper, he first pointed out that, if we take the standpoint of Maxwell's electromagnetic theory and Lorentz's electron theory, the resulting temperature-radiation formula cannot be other than Rayleigh–Jeans' Eq. (8), which contradicted experience. Specifically, according to this formula, the total energy of black-body radiation in a cavity becomes infinite, in contradiction to Stefan–Boltzmann's law.

Then, he proceeded to examine the opposite limit (i.e. large ν/T) to the Rayleigh–Jeans' of the Planck's formula, viz. Wien's radiation formula:

$$\rho = \frac{8\pi h\nu^3}{c^3} e^{-(h\nu)/(kT)}. \tag{9}$$

Utilizing the above, he deduced the entropy of radiation[14] and compared it to the entropies of ideal gases and dilute solutions. The result of the comparison showed that "the entropy of monochromatic radiation of sufficiently low density varies with the volume according to the same law as the entropy of an ideal gas or a dilute solution"[16].

He combined this fact with Boltzmann's principle, which deduced as the probability of finding radiation energy E in the portion v of volume v_0:

$$W = \left(\frac{v}{v_0}\right)^{E/h\nu}. \tag{10}$$

From this he concluded that "monochromatic radiation of low density (within the range of Wien's radiation formula) behaves thermodynamically as if it consisted of mutally independent energy quanta of magnitude $h\nu$" [17].

The paper ends with the indication of three supporting pieces of evidence of this view of 'light-quantum': Stokes' rule in photoluminescence; the generation of cathode rays in the photoelectric phenomena; and the ionization of gases due to the irradiation of ultraviolet light. These phenomena correspond to the following statement in his introduction to this paper [15]:

> [I]t is quite conceivable, despite the complete confirmation of the theory of diffraction, reflection, refraction, dispersion, etc., by experiment, that the theory of light, operating with continuous spatial functions, leads to contradictions when applied to the phenomena of emission and transformation of light.

The decrease in frequency ν of emitted light compared with that of absorbed light in Stokes' rule could be interpreted as a loss of absorbed light energy during the process occurring inside the material. Also, the emission of electrons in the photoelectric phenomena, and in the ionization of gases by the ultraviolet light irradiation, would not take place without a local concentration of light-energy exhibited by the 'light-quantum'.

[14] The application of thermodynamics to black-body radiation was first carried out by Boltzmann in his derivation of Stefan–Boltzmann's law. Following Boltzmann's method, Wien defined the temperature and the entropy of radiation in his paper of 1894.

In the photoelectric phenomena, the light-quantum energy, which penetrates a body's surface layer, can be converted into the kinetic energy of electrons, which subsequently brings about their emission from the body. In leaving the body, each electron has to do some work P. Therefore, the maximum kinetic energy of the emitted electron should be $h\nu - P$.

From this, Einstein proposed his famous photoelectric equation:

$$eV = h\nu - P, \tag{11}$$

where e is the electron charge, V is the positive electric potential of the body raised just enough to prevent the emission of electrons from it. Making use of the approximation $P = 0$, Einstein estimated the value of V from existing data, and obtained "a result that agrees in order of magnitude with the results" of Philipp Lenard's experiment of 1902. Although Lenard's results provided qualitative evidence for Eq. (11), it was not until 1916 that the crucial evidence was provided by Robert Millikan from his experiments.

5. Einstein's First Quantum Theories of Matter and Radiation

It is in his 1906 paper [28] that the full derivation of Planck's radiation formula from Einstein's statistical thermodynamics was presented for the first time. This time, he turns to the collection of resonators, just as Planck did, instead of radiation itself. Einstein showed that, if the energy of resonators is permitted to take a continuous value, the resulting radiation formula is necessarily that of Rayleigh–Jeans' Eq. (8), but if we restrict the energy of resonators to integral multiples of $h\nu$, then Planck's formula (Eq. (9)) results.

The outline of his logic is as follows. As for the relation between the mean energy $\overline{E}$ of resonators with proper frequency ν and energy density ρ of component ν of the radiation, he assumes that Planck's result from the Maxwell's theory holds:

$$\overline{E} = \frac{c^3}{8\pi\nu^2}\rho. \tag{12}$$

In accordance with Planck's method, the expression for $\overline{E}$, and therefore ρ, can be obtained from the thermodynamic relationship $1/T = \partial S/\partial\overline{E}$, if the entropy S of resonators is expressed as a function of $\overline{E}$. In order to accomplish the latter, one can utilize Boltzmann's principle:

$$S = k \log W, \tag{13}$$

where W is the number of divisions of the total energy to each resonator.

Einstein's point of departure from Planck's method is in the expression for W. According to Einstein's statistical thermodynamics discussed in Section 3, W is obtainable from Eqs (3) and (4), by using the relationship

$$\int_{E_\alpha}^{E_\alpha + dE_\alpha} dx_\alpha\, d\xi_\alpha = const \cdot dE_\alpha \tag{14}$$

in terms of the variables describing the resonator α. The result is:

$$W = \int_{H}^{H+\delta H} dE_1 \cdots dE_n, \tag{15}$$

where H is the total energy of n resonators.

If one integrates directly Eq. (15), one gets the equipartition law $\overline{E} = kT$ for the mean energy value per resonator [45], therefore Eq. (12) yields Rayleigh–Jeans' formula Eq. (8) for ρ. On the other hand, it is only when one restricts each value of $E_1, \ldots, E_n$ to integral multiples of $h\nu$, thus reducing the multiple integral to a multiple summation, that one can obtain Planck's result:

$$W = \frac{(n+p-1)!}{(n-1)!p!}, \quad \text{where } p = \frac{H}{h\nu}. \tag{16}$$

That is to say, one obtains Planck's Eq. (9), only when one introduces into the right-hand side of Eq. (14), instead of *const*, the state-density $\omega(E_\alpha)$ of resonator α as a discrete entity (i.e. in the form of a sum of delta-functions, in modern mathematical language).

With regard to this part of Einstein's 1906 paper, Kuhn states [46]:

> That passage is the first public statement that Planck's derivation demands a restriction on the classical continuum of resonator states. In a sense, it announces the birth of the quantum theory.

Thus, in accordance with Kuhn, quantum discontinuity was first introduced in the form of physical theory by Einstein in 1906. At the same time, Einstein's statistical thermodynamics became the first quantum theory, which may have been what Einstein had in mind ever since he began the statistical trio.

Einstein's first quantum theory was readily applied in the following year to the problem of specific heats, which resulted in the 1907 paper [30], where his statistical thermodynamics is applied to the collection of thermal vibrations of atoms in solids. Here, in accordance with what he wrote to Marič in his letter dated 23 March 1901, and cited in Section 1, the vibrations of atoms are treated in the same way as the resonators in the thermal radiation problem.

With the classical continuum of state-density, one obtains the equipartition law for the vibration energy, which yields the Dulong–Petit law[15] for specific heat. According to Einstein's view, however, this law corresponds to Rayleigh–Jeans formula Eq. (8) for radiation. Therefore, the correct specific heat value corresponding to Planck's law, Eq. (9), could only be obtained by introducing discreteness into the state-density of the vibrating atoms. The temperature dependence of the value of specific heat, thus obtained by Einstein, shows that, irrespective of the species of matter, the specific heat value should come nearer to zero as the temperature goes to zero.

Walther Nernst, professor of physical chemistry at the University of Berlin, was astonished at the agreement of Einstein's theory with his experimental results, especially with the 'Nernst's theorem' concerning the temperature dependence of thermodynamic quantities in the low temperature region. Therefore, he convened the first Solvay conference on 'Radiation Theory and Quanta' in 1911. It was after this conference that contemporary physicists came to consider seriously the theory of quanta. The proceedings

[15] Dulong–Petit's law states that the product of the specific heat [cal/g K] and the atomic weight [g/mol] is constant for different simple solids. This law was discovered by P.L. Dulong and A.T. Petit in 1819, and was utilized to determine the atomic weight of various elements.

of this conference [50] left a lasting impression on physicists, two fruits of which were Niels Bohr's theory of atomic structure in 1913 [5] and Louis de Broglie's theory of matter waves in 1923 [10].

At the conference, after his lecture on "The Present State of the Problem of Specific Heat", Einstein stated [34]:

> [I]t also turned out that classical mechanics [...] can no longer be viewed as a useful schema for theoretical representation of all of physical phenomena. This raises the question of which general laws of physics we can still expect to be valid in the domain with which we are concerned. To begin with, we will all agree that the energy principle is to be retained. In my opinion, another principle whose validity we must maintain unconditionally is Boltzmann's definition of entropy through probability.

As the two laws Einstein referred to above correspond to the first and the second laws of thermodynamics, this statement confirms my conjecture that the core ingredient of Einstein's research programme was thermodynamics. It may also clearly explain the reason why he constructed the statistical thermodynamics of 1903–1904, aiming at the first quantum theory.

Einstein discussed 'Nernst's theorem' again in 1914 paper [12], where he treated a collection of molecules carrying one resonator each. What is interesting about this paper is that the chemical distinction between molecules is reduced to the difference in discrete energy values that the resonators carried by them can take. Moreover, as will be discussed in Section 7, his concern with Nernst's theorem (later known as 'the third law of thermodynamics') led him to the quantum theory of ideal gas in 1924–1925 [31].

Having finished the construction of the general theory of relativity, in 1916 Einstein returned to the problem of thermal radiation. The resultant paper [13] consists of three parts, the first of which is entitled "Planck's Resonator in a Field of Radiation". In this part, Einstein distinguished two kinds of energy change taking place in resonators over a short time interval, the first of which is due to the emission of radiation, and the second is due to the work done by the electric field of the incident radiation on the resonator.

The second part of the paper is entitled "Quantum Theory and Radiation". This section treats a collection of identical molecules in static equilibrium with thermal radiation. In accordance with Bohr's theory of 1913, each molecule is assumed to be in discrete energy states. It should be remarked here that, whereas Bohr and his colleagues, e.g. Arnold Sommerfeld, treated the electronic motion within a single atom, Einstein treated a collection of molecules in thermal equilibrium. Here we can see a clear difference in their approaches to quantum theory, which brought forth their subsequent disagreement about the character of quantum theory. We will return to this problem in the final section of this chapter.

In a somewhat similar manner to the 1914 paper, Einstein assumed that the molecules interact with radiation in just the same way as resonators do. Corresponding to the first kind of energy change of the resonator, he put the number of transitions per unit time from higher energy state m to lower n as $A_m^n N_m$, where N_m is the number of molecules in the state m. Corresponding to the second kind, he put the number of transitions per unit time from lower state n to higher m, and from higher m to lower n, as $B_n^m N_n \rho$ and $B_m^n N_m \rho$ respectively, with ρ being the radiation energy density. The values of A_m^n and $B_m^n \rho$ are identified, in the next paper to be discussed below, as the probability of transition from state m to n per unit time per atom. Therefore, we can see that the concept of 'transition probability' was first introduced into physics by Einstein in these papers.

Assuming the statistical equilibrium between the transitions from m to n and from n to m, i.e.

$$A_m^n N_m + B_m^n N_m \rho = B_n^m N_n \rho, \tag{17}$$

and assigning to the relative values of N_m and N_n their probabilities at thermal equilibrium with constant temperature T,

$$\frac{N_n}{N_m} = \frac{p_n}{p_m} e^{(\varepsilon_m - \varepsilon_n)/(kT)}, \tag{18}$$

he could derive the same type of relationship between ρ and T as in Planck's Eq. (9). Applying Wien's displacement law to this relationship, he could also obtain Bohr's frequency condition $\varepsilon_m - \varepsilon_n = h\nu$.

At this point, Einstein remarked [13, pp. 215–216]:

> *[T]he natural connection to Planck's linear oscillator (as a limiting case of classical electrodynamics and mechanics)* seem to make it highly probable that these are basic traits of a future theoretical representation. [Italics added]

This remark is believed to be the root of Bohr's 'correspondence principle' mentioned in Bohr's 1918 paper on the atomic spectra, where this paper was cited [6].

Einstein also pointed out that the first kind of energy change is simply Rutherford's law of radioactive decay. Therefore, the first kind of change (later called 'spontaneous emission') is regarded as corresponding to the particle-like behavior of radiation. On the other hand, the second kind of change (later called 'induced emission and absorption'), which derives from the resonator's interaction with the electric wave of radiation, is regarded as corresponding to the wave-like property of radiation. Thus, the deduction of Planck's formula in this paper seems to be based on reversed inference from the fact that Planck's formula leads both to particle-like and wave-like behaviour in the fluctuation of thermal radiation, with which the molecules are kept at equilibrium.

The last part of this paper is entitled "Remark on the Law of Photochemical Equivalence". The latter law, first expounded in his 1912 paper [33], means that the decomposition of one molecule in a photochemical process correponds to the absorption of one light quantum $h\nu$.

In another 1916 paper, entitled "On the Quantum Theory of Radiation" (the same paper was published again in 1917), Einstein posed the question [27]:

> Does the molecule receive an impulse when it absorbs or emits energy ε? For example, let us look at emission from the point of view of classical electrodynamics. When a body emits energy ε it suffers a recoil (momentum) ε/c if the entire amount of radiation energy ε is emitted in the same direction.

He answered this question himself as follows [27, pp. 221–222]:

> *It turns out that we arrive at a theory that is free of contradictions, only if we interpret those elementary processes as completely directed processes.* Here lies the main result of the following considerations. [Einstein's italics]

Here, Einstein treated the motion of molecules immersed in thermal radiation. In the same type of reasoning as that used in the Brownian movement of a suspended mirror in his 1909 paper, he derived the same relationship as that obtainable by combining Eqs (6) and (7). He calculated both sides of this equation independently, utilizing the quantum theory of radiation of the preceding paper. Calculations of the left-hand side (i.e. $\overline{\Delta^2}$) was done in the rest coordinate system, while the right-hand side (i.e. P) was done in the moving coordinate system by utilizing special relativity.

The result of the calculations showed that it is only when one allows the association of momentum transfer $(\varepsilon_m - \varepsilon_n)/c$ that both sides of the equation agree. In this way, Einstein showed that his 'light quantum' carries, not only energy $h\nu$, but also momentum $\frac{h\nu}{c}$, thus reducing the 'light quantum' to the modern concept of a photon. However, the acceptance of this concept by the wider physical community required the discovery of the Compton effect in 1923.

6. Interlude: Two Research Traditions in Physics at the Turn of the Century

Usually, modern science, especially physics, is regarded as constructed during the 'Scientific Revolution' that took place during the sixteenth and seventeenth centuries. However, it is also well known that, during the nineteenth century, modern science underwent another transformation, due to the Industrial Revolution of the eighteenth and nineteenth centuries and the accompanying institutionalization of science, i.e. the establishment of various scientific communities and the emergence of professional scientists. Stephen Brush, famous historian of science, termed this process of the transformation of science during the nineteenth century and the first half of the twentieth century "the second scientific revolution" [9].

Among the transformations that took place in science, the most conspicuous was the emergence of new scientific disciplines such as chemistry, thermodynamics, electromagnetism, and physiology. Although the pioneers in these new fields were British technologists/scientists during the Industrial Revolution, and French during the French revolution, systematic scientific research in these new fields was begun by German scientists in the nineteenth century. The term 'physics' (Physik) itself, as the name of a subject, was introduced during the German university reformation in the early nineteenth century [51]. During the latter half of that century, as a result of university reformation, research universities, which carried out research in the new scientific fields, were built all over Germany.

In this process, the unification of experimental and theoretical research was also pursued. Another famous historian of science, Russell McCormmach, once stated [52]:

> Early in the nineteenth century, German mathematical physics, rational mechanics, and applied mathematics tended to be practiced by people trained in mathematics; and experimental physics tended to be practiced by those trained in chemistry. The two groups published in different places and worked in distinct, essentially noncommunicating sciences. Many early organizers of German physics discipline such as Johann Christian Poggendorff and Heinrich Gustav Magnus strongly condemned the infusion of mathematical theory into physics. [...] In the l830's and 1840's the division between mathematicians and experimentalists became less sharp, due in part to the development of a method of mathematical physics that drew attention to the common purposes of the two groups.

I have called this largely German, mathematical-experimental tradition of the later nineteenth century 'the chemico-thermal tradition' [1]. The reason I used the term "chemico-" here is that the members of this tradition were often interested, as their roots might suggest, in the chemical phenomena, especially of physical chemistry.

The main body or scientific community of this tradition can be identified with the Helmholtzian School, which consisted of colleagues, students, and followers of Hermann von Helmholtz at the Berlin Physical Society, the universities of Heidelberg and Berlin, and the Physical-Technological Imperial Institute. Among its celebrated members were Clausius, Kirchhoff, Helmholtz, Hertz, Wien, Boltzmann, Planck, and Ernst Mach, most of whom studied hydrodynamics, the kinetic as well as thermodynamical gas theory, chemical thermodynamics, the theory of thermal radiation, and the electromagnetic-field theory. Some of them, especially Kirchhoff, Mach, and Hertz, engaged in the 'criticism of mechanics' and tried to reformulate mechanics into an empirical theory based primarily on the picture of a continuous medium. From my point of view, the famous debate between atomistics and energetics, which concerned the foundations of thermodynamics and its relationship with other branches of physics, was an intra-school controversy.

Nothing quite like the German chemico-thermal tradition existed elsewhere. The separation of physical sciences into mathematical and experimental continued in Britain, France, and Holland. Men like John Rayleigh, James Jeans, Henri Poincaré, and Hendrik Lorentz worked mainly in the classical sciences [48], i.e. in astronomy, harmonics, mathematics, optics and dynamics. Nevertheless, after the discovery of cathode rays, mathematical and experimental physicists gradually began to interact in these countries.

The style of interaction, however, was very different from that in Germany. Unlike the Germans, they worked together by infusing experimental methods into the classical sciences. These attempts met with great success: in 1892, Lorentz offered his electron theory, in 1896 Pieter Zeeman discovered the effect named after him, in 1897 Joseph J. Thomson identified the electron, and in 1911 Ernest Rutherford identified the atomic nucleus and constructed his atomic model. I shall call this line of research 'the particle-dynamical tradition' and identify its main body or scientific community as the Cavendish School.

It would be wrong to suppose that the particle-dynamical tradition had no adherents in Germany. In Germany there were groups of physicists and mathematicians who concentrated solely on the electron theory. Emil Wiechert, Max Abraham, Sommerfeld, and the mathematical physicists of the Göttingen electron-theory seminar may be counted as members of the particle-dynamical tradition.

Generally speaking, the chemico-thermal tradition constituted the core of the second scientific revolution, and scientists in this tradition placed great importance on the concept of entropy. While the particle-dynamical tradition inherited much of the legacy of the (first) Scientific Revolution from the time of Newton, the scientists in this tradition treasured the concept of the ether. In this division, Einstein stood within the chemico-thermal tradition. In contrast, Bohr, who studied under Rutherford, can be regarded as a member of the particle-dynamical tradition. This point may be relevant to the later controversy between Einstein and Bohr over the evaluation of quantum mechanics.

7. Einstein's Contributions to Early Quantum Theory

Very often, the history of early quantum theory is described as a refining process of Bohr's theory of atomic structure. But, as Martin J. Klein put it [44], that picture of the development "lacks just that variety and complexity peculiar to the principal achievement of twentieth century physics". As we have seen, the wave–particle duality, the most peculiar to the quantum theory, was first introduced by Einstein. Moreover, he was the first to introduce the energy quantization of matter, expressed by the introduction of the discreteness in the state-density of his statistical thermodynamics, thus constructing the first quantum theory in 1906.

His construction of the quantum theory of radiation introduced the germ of the so-called correspondence principle. Also, in that theory, the concept of transition probability was introduced in connection with the concept of resonators being carried by molecules. The former concept (transition probability) inspired Werner Heisenberg to construct his 'matrix mechanics' in 1925, while the latter concept (resonators carried by molecules) may be the origin of Hendrik A. Kramers' virtual oscillator model (he was working with Heisenberg at that time).

On the other hand, Einstein's first quantum theory was connected, through its application to the problem of specific heats, with Nernst's theorem, i.e. the third law of thermodynamics. The requirement of the latter theorem, as well as the avoidance of Gibbs' paradox (i.e. the production of 'entropy of mixture'[16] by mixing the same type of gas), led Einstein to construct the quantum theory of the ideal gas in 1924–1925, when he applied to the ideal gas the same statistics that Satyendra N. Bose applied to thermal radiation. This theory stimulated Erwin Schrödinger to look seriously into de Broglie's theory of matter waves, which subsequently led Schrödinger to construct his wave mechanics in 1926.

The above overview suggests that the most important steps in the construction of quantum mechanics were made, not by Bohr, but by Einstein. As I noted in the previous section, Bohr's attention was concentrated on internal electronic processes in atoms and molecules, supplemented by some philosophical considerations thereupon. This peculiarity must be closely related to his early studies in Britain at Thomson's and Rutherford's laboratories.

On the other hand, Einstein's concern spreads so wide from the physical chemistry to the space–time structure that his aim was to construct a consistent theory that was sustainable from every point of view. He also frequently discussed chemical and thermal effects in the material sciences. He inherited these concerns from reading books and articles by Kirchhoff, Helmholtz, Hertz, Wien, Boltzmann and Planck, in what I called the 'chemico-thermal tradition' in the last section.

As is pointed out in Chapter 4, even his attempt at the special theory of relativity seems not unrelated to his aim of constructing quantum theory. Moreover, he had in mind up to the end of his life the hope of constructing his own theory of the microscopic world from his unified field theory. Therefore, even his construction of the general theory of relativity itself might have been a step towards constructing his own microscopic theory, which was after all his lifelong aim throughout his scientific career [41].

8. Postlude: Einstein's Attitude toward Quantum Mechanics

Heisenberg, one of the pioneers of quantum mechanics, once commented on Einstein's attitude to it [38]:

[16]The 'entropy of mixture' is defined as the difference between the sum of the entropies of two separated substances and the entropy of their mixture.

> In the course of scientific progress it can happen that a new range of empirical data can be completely understood only when enormous effort is made to change one's philosophical framework and to change the very structure of the thought process. In the case of quantum mechanics, Einstein was no longer willing to take this step, or perhaps no longer able to do so.

This kind of view was shared among the scientists of the Copenhagen school, i.e. the colleagues, students and followers of Bohr. Contrary to that, Arthur Fine, famous philosopher of science, expressed the view [37]:

> It is Bohr who emerges the conservative, unwilling (or unable?) to contemplate the overthrow of the system of classical concepts and defending it by recourse to those very conceptual necessities and *a priori* arguments. [. . .] Whereas, with regard to the use of classical concepts, Einstein's analytical method kept him ever open-minded.

In the fifth (1927) and the sixth (1930) Solvay conferences, Einstein and Bohr exchanged discussions on their views of quantum mechanics. This discussion is described in Bohr's memoirs expressing the latter's triumph [4].

Seen from Einstein's side, however, the process was very different. According to the notes enclosed in a letter to Lorentz after the fifth conference, Einstein argued there [65]:

> [I]f the state function were interpreted as expressing probabilities for finding properties of an individual system, then the phenomenon of the collapse of the wave packet would represent a peculiar action-at-a distance. [. . .] It represents a peculiar nonlocalized mechanism, which violates relativity. [. . .] These problems are not of the theory itself but of the interpretation according to which the theory gives a complete statistical description of individual systems. The alternative is to interpret the state function as providing information only about the distribution of an ensemble of systems and not about features of the individual system themselves.

Also, in a letter to Sommerfeld just after the fifth conference, he wrote [39]:

> On 'Quantum Mechanics' I think that, with respect to ponderable matter, it contains roughly as much truth as the theory of light without quanta. It may be a correct theory of statistical laws, but an inadequate conception of individual elementary processes.

During these two conferences, Bohr rebutted Einstein's argument using the doctrine of disturbance, stating that certain simultaneous determinations were not possible because any one of them would inevitably disturb the physical situation so as to preclude the other. Therefore, the purpose of the EPR paper [11], i.e. Einstein's paper of 1935 co-authored with Boris Podolsky and Nathan Rosen, was to neutralize this doctrine of disturbance. On the essence of this paper, Einstein wrote to Schrödinger [64]:

> Consider a ball located in one of two boxes. An incomplete description of this 'reality' might be 'The probability is one-half that the ball is in the first box'. A complete description would be 'The ball is in the first box'. Assuming a principle of separation, i.e. 'the contents of the second box are independent of what happens to the first box', and an obvious conservation law for the number of ball, one can find out by looking in the first box whether or not the ball is in the second box. If a theory only allows, in these circumstances, probabilistic assertions, that theory is incomplete.

Fine commented on the EPR paper, "I think it is fair to conclude that the EPR paper did succeed in neutralizing Bohr's doctrine of disturbance" [35].

The standard statistical interpretation using ensemble and hidden parameters has been confronted by difficulties. In 1964, John Bell showed that statistical interpretation is actually in numerical inconsistency with quantum theory when applied to a coupled system treated in the EPR paper [3].

According to Fine, however, the interpretation Einstein entertained was not the standard one but one described by the prism model, in Fine's terminology. He drew attention to a footnote in Einstein's 1936 article [29]:

> A measurement on A, for example, thus involves a transition to a narrower ensemble of systems. The latter (hence also its ψ function) depends upon the point of view according to which the reduction of the ensemble of systems is carried out.

Starting from this footnote, Fine constructed his prism model theory representing Einstein's idea. According to Fine, this model produces the same results as does the quantum mechanics, and thus does not conflict with Bell's theorem [36].

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