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Robust H-infinity State Estimation of Uncertain Neural Networks with Two Additive Time-Varying Delays

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Abstract. This study is mainly concerned with the problem of robust *H*∞ state estimation of uncertain neural networks with two additive time-varying delays. A novel linear matrix inequalities (LMIs) is constructed based on Lyapunov-Krasovskii functionals (LKFs) which contains two additive time-varying delays components. LMIs method are used to estimate the derivative of LKFs, it is calculated that the derivative of the LKFs is smaller than zero, which proved that uncertain neural networks with two additive time-varying delays is globally asymptotically stable. Meantime, a stability criterion of error system is presented such that the H∞ performance is guaranteed. Finally, two numerical simulation examples have been performed to demonstrate the effectiveness of developed approach.

Keywords. linear matrix inequalities (LMIs), *H*∞ state estimation, two time-varying delays, uncertain neural networks

1. Introduction

Time-varying delays has received much attention which exists in many industrial and engineering systems [1, 2]. Time-varying delays frequently cause oscillations, divergence or instability of the systems, so the systems stability is the main consideration for time-varying delay in many applications.

The stability problems for time-varying delays systems have been researched in recent years. For example, Wu [3] studied the stability of an uncertain systems, which contains multiple consecutive delay components. By considering the relationship between the time-varying delays and its upper bound, but there is no system state estimation, it is difficult to discover the change of the system, and the random interference items has not been added. Whether the delay is included according to the stability criteria, the delay systems is divided into two classes by Liu [4] which are delay-independent and delay-dependent. Two additive time-varying delays system is studied by Xiong [5] and establishes two novel integral inequalities, but the calculation process of this research is too cumbersome. The stability of continuous linear system with two additive time-varying delays is studied by Xu [6], which utilized the

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reciprocally convex combination technique. Subramanian [7] and Samidurai [8] studied robust analysis for uncertain neural networks with two additive time-varying delays by constructing Lyapunov-Krasovskii functions (LKFs) and making use of linear matrix inequalities (LMIs) technique. Liang [9] and Yuan [10] studied complex-valued neural networks (CVNNs) with two additive time-varying delays by constructing LKFs and making use of LMIs to solve the stable problems. The results show the convergence of the real and imaginary parts. The above literatures are for the stability analysis of two additive time-varying delays system. These literatures did not refer to the system by constructing state estimator.

Bao [11]. Hou [12], Liu[13], Shen [14], Zhang [15] and Zhao [16] studied with $H\infty$ state estimation problem for time-varying artificial neural networks. The aim of these papers is to design a time-varying $H\infty$ estimator, such that the dynamics of the estimation error satisfy the given $H\infty$ performance requirement. Liu [17] studied the problem of $H\infty$ state estimation of a static neural network with time-varying delay by constructing a suitable Lyapunov function and ensuring the attenuation of the results as early as possible, which indicates that the system is progressively stable under Lyapunov's conditions. $H\infty$ state estimation had been constructed in above literatures. Shao [18] studied an $H\infty$ control problem with delay-dependent stability condition, a new stability criteria is obtained, and finally the system is proved to be asymptotically stable by constructing a Lyapunov functional. Zhou [19] studied the robust $H\infty$ control problem for a delay singular system with parameter uncertainties. The influence of the system on the disturbance is attenuated, which indicate that the system is in the state of Lyapunov asymptotically stable by constructing a robust state feedback control law. A non-fragile state estimator of the recurrent delayed neural networks is designed by Yang [20] to ensure the existence of the desired estimators. The inadequacy of these researches is that they do not consider two additive time-varying delays issues. Among them, the $H\infty$ control essence is an optimization design using the $H\infty$ norm as the objective function. The $H\infty$ norm is a norm defined on the Hardy space. In the $H\infty$ control theory, it refers to the maximum singular value of the rational function matrix parsed in the right half plane of S.

Zhou [21] studied the problem of robust finite-time state estimation for a class of discrete-time neural networks with two delay components and Mrakovian jump parameters. And a new LKFs is constructed. Duan [22] investigated the state estimation for $H\infty$ control static neural network with two additive time delays. Time-varying delays often occur in engineering systems, network control, and biological network control. It is a factor that must be considered in the practice problems. In the problems of robust control and nonlinear asymptotically stable control, time-varying delays are factors that must be considered [23]. The difference between the leakage delays and the time-varying delays is that it will cause the instability of the system, and the time-varying delays can cause system delays [24].

There is less research in uncertain neural networks with two additive time-varying delays, in this paper, we investigate the $H\infty$ state estimation for uncertain neural networks with two additive time-varying delays. Based on LKFs method, a novel LMIs method has been established to ensure the global asymptotic stability of uncertain neural networks with two additive time-varying delays. Finally, two numerical simulation examples are used to illustrate the effectiveness of the proposed design method.

2. System model and Preliminaries

Considering the following uncertain neural networks with two additive time-varying delays:

$$\begin{cases} \dot{x}(t) = -A_1(t)x(t) + A_2(t)f(x(t)) + A_3(t)x(t-\tau_1(t)-\tau_2(t)) + C_1(t)w(t) + u(t) \\ y(t) = B_1(t)x(t) + B_2(t)x(t-\tau_1(t)-\tau_2(t)) + D_1(t)w(t) \\ z(t) = Hx(t) \\ x(t) = \varphi(t), t \in [-h, 0] \end{cases} \quad (1)$$

Where $x(t) \in [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \Re^n$ is the state vector, $y(t)$ is the measurement, $u(t)$ is the control input, $z(t) \in$ to be estimated is a linear combination of the state, $w(t)$ is disturbance input and $w(t) \in L^2[0,\infty]$, and A_1, A_2, A_3, C_1, B_1, B_2, D_1 and H are known real constant matrices, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)) \ldots\ldots f_n(x_n(t))]^T$ denotes the neuron activation function and a constant input vector, and. $A_1(t) = A_1 + \Delta A_1$, $A_2(t) = A_2 + \Delta A_2$, $A_3(t) = A_3 + \Delta A_3$, $B_1(t) = B_1 + \Delta B_1$, $B_2(t) = B_2 + \Delta B_2$, $C_1(t) = C_1 + \Delta C_1$, $D_1(t) = D_1 + \Delta D_1$, ΔA_1, ΔA_2, ΔA_3, ΔB_1, ΔB_2, ΔC_1 and ΔD_1 is real matrix. These parameters represent the uncertainty of the system. $\varphi(t)$ is represents a given initial condition.

Assumption 1. The parameter uncertainties ΔA_1, ΔA_2, ΔA_3, ΔB_1, ΔB_2, ΔC_1 and ΔD_1 are of the form:

$$[\Delta A_1 \quad \Delta A_2 \quad \Delta A_3 \quad \Delta B_1 \quad \Delta B_2 \quad \Delta C_1 \quad \Delta D_1] = HF(t)[M_{11} \quad M_{12} \quad M_{21} \quad M_{22} \quad S_1 \quad S_2 \quad S_3] \quad (2)$$

Among these parameters, $H, M_{11}, M_{12}, M_{21}, M_{22}, S_1, S_2, P_1$ is known real matrix of appropriate dimension. $F(t) \in R^{i \times j}, i > 0, j > 0$ is the real unknown time-varying matrix. So,

$$F(t)^T F(t) \le I, \forall t \ge 0 \quad (3)$$

Assumption 2. The time-varying delays $\tau_1(t), \tau_2(t)$ satisfy

$$0 \le \tau_1(t) \le \tau_1 \le \infty, 0 \le \tau_2(t) \le \tau_2 \le \infty \quad (4)$$

$$\dot{\tau}_1(t) \le \mu_1, \dot{\tau}_2(t) \le \mu_2 \quad (5)$$

$$\tau = \tau_1 + \tau_2, \mu = \mu_1 + \mu_2 \quad (6)$$

where τ_1, τ_2 are positive constants.

Assumption 3 [25]. Each neuron activation function $f_i(.), i = 1,2,\ldots,n$ satisfies the following condition:

$$0 \le \frac{f_i(\alpha) - f_i(\beta)}{\alpha - \beta} \le l_i, \forall \alpha, \beta \in R, \alpha \ne \beta \quad (7)$$

where $l_i, i = 1,2,\ldots,n$ are constants, and diagonal matrix $L = diag\{l_i\}$.
Then, constructing a state estimator for estimation of $Z(t)$ as follows

$$\begin{cases} \dot{\hat{x}}(t) = -A_1(t)\hat{x}(t) + A_2(t)f(\hat{x}(t)) + A_3(t)\hat{x}(t-\tau_1(t)-\tau_2(t)) \\ \qquad + K(y(t) - B_1(t)\hat{x}(t) + B_2(t)\hat{x}(t-\tau_1(t)-\tau_2(t))) + u \\ \hat{z}(t) = H\hat{x}(t) \\ x(t) = 0, t \in [-h, 0] \end{cases} \tag{8}$$

where $\hat{x}(t) \in \mathbb{R}^n$ denotes the estimated state, $\hat{z}(t) \in \mathbb{R}^n$ denotes the estimated measurement of $z(t)$, and K is the state estimator gain matrix.

Denoting the errors by $e(t) = x(t) - \hat{x}(t)$ and $\tau(t) = \tau_1(t) + \tau_2(t)$, $\bar{z}(t) = z(t) - \hat{z}(t)$, $e(t-\tau_1(t)-\tau_2(t)) = x(t-\tau_1(t)-\tau_2(t)) - \hat{x}(t-\tau_1(t)-\tau_2(t))$

Then, based on (1) and (8), the error system of the form is obtained:

$$\begin{cases} \dot{e}(t) = -(A_1(t) + KB_1(t))e(t) + A_2(t)g(e(t)) + (A_3(t) - KB_2(t))e(t-\tau_1(t)-\tau_2(t)) \\ \qquad + (C_1(t) - KD_1(t))w(t) \\ \bar{z}(t) = He(t) \end{cases} \tag{9}$$

Where $g(e(t)) = f(x(t)) - f(\hat{x}(t))$,

$$\begin{cases} \dot{e}(t) = -(A_1(t) + KB_1(t))e(t) + A_2(t)g(e(t)) + (A_3(t) - KB_2(t))e(t-\tau(t)) \\ \qquad + (C_1(t) - KD_1(t))w(t) \\ \bar{z}(t) = He(t) \end{cases} \tag{10}$$

In this paper we will study the stability of (10) so that guaranteed its $H\infty$ performance. Moreover, it is proved by numerical simulation that the state estimation of the system error equation tends to zero, which proves that the system is asymptotically stable.

Definition 1 [25]**.** Given a prescribed level of noise attenuation $\gamma \geq 0$, a proper state estimator (8) is founded, the equilibrium point of the result error system (10) with $w(t) = 0$ is globally asymptotically stable, and

$$\|\bar{z}(t)\|_2 < \gamma \|w(t)\|_2 \tag{11}$$

under zero-initial conditions for all nonzero $w(t) \in L_2[0, \infty)$, where

$$\|x(t)\|_2 \triangleq \sqrt{\int_0^\infty x^T(t)x(t)dt} \tag{12}$$

In this case, error system (10) is globally asymptotically stable with $H\infty$ performance γ.

3. Results and Proof

In this section, the global asymptotic stability of the model (10) with the initial condition (2)~(7) is discussed and the main results are given as follows:

Theorem 1. Considering the uncertain neural networks with two additive time-varying delays (1), If there is an appropriate dimension of the matrix Y, for given scalars $h_1, h_2 > 0$, let γ to be a prescribed constant, then the guaranteed $H\infty$ performance state estimation problem is solvable, if there exist real matrices $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 > 0$ $T > 0, P_1 > 0, R_i > 0, S_i > 0 (i = 1,2,3), M_{je} > 0 (j = 1,2, e = 1,2);$ positive diagonal matrices $P_2 = diag\{\eta_i\}$, so the following LMIs are satisfied:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} & \Omega_{28} & \Omega_{29} & \Omega_{210} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} & \Omega_{38} & \Omega_{39} & \Omega_{310} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} & \Omega_{47} & \Omega_{48} & \Omega_{49} & \Omega_{410} \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{1010} \end{bmatrix} < 0 \quad (13)$$

where

$\Omega_{11} = -A_1P_1 + B_1Y - A_1^TP_1^T + B_1^TY^T + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + M_{11}^T + M_{11} + M_{12}^T + M_{12} - T + H^TH$, $\Omega_{1010} = -\gamma^2 I$
$\Omega_{12} = A_3P_1 - B_2Y - A_1^TP_1^T + B_1^TY^T + R_2 - T - M_{11}^T + M_{21}$, $\Omega_{13} = A_2P_1 - A_1^TP_1^T + B_1^TY^T - M_{12}^T + M_{22}$, $\Omega_{14} = R_2$,
$\Omega_{15} = A_2P_2 - A_1^TP_1^T + B_1^TY^T$, $\Omega_{16} = \tau_1(A_3^TP_1^T + B_1^TY^T) - A_1^TP_1^T + B_1^TY^T$, $\Omega_{17} = \tau_2(A_3^TP_1^T + B_1^TY^T) - A_1^TP_1^T + B_1^TY^T$, $\Omega_{18} = \tau_1S_1$
$\Omega_{19} = \tau_2S_2$, $\Omega_{110} = P_1C_1$, $\Omega_{22} = A_1P_1 - (1-\tau)M_{11}^T + (1-\tau)M_{22}$, $\Omega_{23} = 0$, $\Omega_{24} = R_2 - T$, $\Omega_{25} = (A_1P_1)^T$, $\Omega_{26} = \tau_1(A_1P_1)^T$,
$\Omega_{27} = \tau_2(A_1P_1)^T$, $\Omega_{28} = \tau_2S_3$, $\Omega_{29} = 0$, $\Omega_{210} = 0$, $\Omega_{33} = -Q_2 - (1-\tau)M_{11}^T + (1-\tau)M_{22}$, $\Omega_{34} = 0$, $\Omega_{35} = (B_1P_1)^T$,
$\Omega_{36} = \tau_1(B_1P_1)^T$, $\Omega_{37} = \tau_2(B_1P_1)^T$, $\Omega_{38} = 0$, $\Omega_{39} = \tau_2R_3$, $\Omega_{310} = 0$, $\Omega_{44} = -P_1$, $\Omega_{45} = B_1^T$ $\Omega_{46} = B_2^T$, $\Omega_{47} = \tau_2B_1^T$,
$\Omega_{48} = \tau_2B_2^T$, $\Omega_{49} = 0$, $\Omega_{410} = D_1$, $\Omega_{55} = -\tau R_3$, $\Omega_{66} = -\tau_1R_1$, $\Omega_{77} = -\tau_2R_2$, $\Omega_{88} = -\tau_1R_1^T$, $\Omega_{99} = -\tau_2R_2^T$,
where $K = P^{-1}Y$

Proof 1. constructing the following Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \quad (14)$$

$$V_1(e_t) = e^T(t)P_1e(t) + 2\sum_{i=1}^{n} P\int_0^{e_i(t)} g(s)ds \quad (15)$$

$$V_2(e_t) = \int_{t-\tau_1(t)}^{t} e^T(s)Q_1e(s)ds + \int_{t-\tau_2(t)}^{t} e^T(s)Q_2e(s)ds + \int_{t-\tau(t)}^{t} e^T(s)Q_3e(s)ds + \int_{t-\tau_1}^{t} e^T(s)Q_4e(s)ds + \int_{t-\tau_2}^{t} e^T(s)Q_5e(s)ds + \int_{t-\tau}^{t} e^T(s)Q_6e(s)ds \quad (16)$$

$$V_3(e_t) = \int_{t-\tau(t)}^{t} \begin{bmatrix} e(s) \\ g(e(s)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} e(s) \\ g(e(s)) \end{bmatrix} ds, \quad (17)$$

$$V_4(e_t) = \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} e^T(s)R_1e(s)dsd\theta + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} e^T(s)R_2e(s)dsd\theta + \tau_2\int_{-\tau_2}^{0}\int_{t+\theta}^{t} e^T(s)R_3e(s)dsd\theta \quad (18)$$

Taking the time-derivative of $V(t)$ along the trajectories of yields. Robust stabilization of the system. In this case, error system (10) is globally asymptotically stable with $H\infty$ performance γ .

$$\dot{V}_1(e(t)) = 2e^T(t)P_1\dot{e}(t) + 2g^T(\mathrm{e}(t))P_2\dot{e}(t) \tag{19}$$
$$= 2e^T(t)P_1[-(A_1(t)+KB_1(t))e(t) + A_2(t)g(e(t)) + (A_3(t)-KB_2(t))e(t-\tau(t))$$
$$+(C_1(t)-KD_1(t))w(t)] + 2g^T(\mathrm{e}(t))P_2[-(A_1(t)+KB_1(t))e(t) + A_2(t)g(e(t))$$
$$+(A_3(t)-KB_2(t))e(t-\tau(t)) + (C_1(t)-KD_1(t))w(t)]$$

$$= -2e^T(t)P_1(A_1(t)+KB_1(t))e(t) + 2e^T(t)P_1(A_3(t)-KB_2(t))e(t-\tau(t)) + 2e^T(t)P_1A_2(t)g(e(t))$$
$$+2e^T(t)P_1(C_1(t)-KD_1(t))w(t) - 2g^T(\mathrm{e}(t))P_2(A_1(t)+KB_1(t))e(t) + 2g^T(\mathrm{e}(t))P_2(A_3(t)-KB_2(t))e(t-\tau(t))$$
$$+2g^T(\mathrm{e}(t))P_2A_2(t)g(e(t)) + 2g^T(\mathrm{e}(t))P_2(C_1(t)-KD_1(t))w(t)$$

$$\dot{V}_2(e(t)) \le e^T(t)(Q_1+Q_2+Q_3+Q_4+Q_5+Q_6)e(t) - e^T(t-\tau_1)Q_1e(t-\tau_1) \tag{20}$$
$$-e^T(t-\tau_2)Q_2e(t-\tau_2) - e^T(t-\tau)Q_3e(t-\tau) - (1-\mu_1)e^T(t-\tau_1(t))Q_4e(t-\tau_1(t))$$
$$-(1-\mu_2)e^T(t-\tau_2(t))Q_5e(t-\tau_2(t)) - (1-\mu)e^T(t-\tau(t))Q_6e(t-\tau(t))$$

$$\dot{V}_3(e(t)) = \begin{bmatrix} e(t) \\ g(e(t)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} e(t) \\ g(e(t)) \end{bmatrix} - (1-\dot{\tau}(t)) \begin{bmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{bmatrix} \tag{21}$$
$$\le e^T(t)R_1e(t) + g^T(e(t))R_3g(e(t) + 2e^T(t)R_2g(e(t)) - (1-\mu)e^T(t-\tau(t))R_1e(t-\tau(t))$$
$$-(1-\mu)g^T(\mathrm{e}(t-\tau(t))R_3g(\mathrm{e}(t-\tau(t))) - 2(1-\mu)e^T(t-\tau(t))R_2g(\mathrm{e}(t-\tau(t)))$$

$$\dot{V}_4(e(t)) = \tau_1^2\dot{e}^{\mathrm{T}}(t)R_1\dot{e}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{e}^{\mathrm{T}}(s)R_1\dot{e}(s)ds + \tau_2^2\dot{e}^T(t)R_3\dot{e}(t) - \tau_2\int_{t-\tau_2}^{t}\dot{e}^T(s)R_3\dot{e}(s)ds \tag{22}$$
$$+\tau^2\dot{e}^T(t)R_2\dot{e}(t) - \tau\int_{t-\tau}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds$$

Among the equation of the $\dot{V}_4(e(t))$, Then $\dot{V}_4(e(t))$ is obtained through above formulas:

$$\dot{V}_4(e(t)) \le \tau_1^2\dot{e}^T(t)R_1\dot{e}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{e}^T(s)R_1\dot{e}(s)ds + \tau_2^2\dot{e}^T(t)R_3\dot{e}(t) - \tau_2\int_{t-\tau_2}^{t}\dot{e}^T(s)R_3\dot{e}(s)ds$$
$$+\tau^2\dot{e}^T(t)R_2\dot{e}(t) - \tau\int_{t-\tau}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \tag{23}$$
$$= \tau^2\dot{e}^T(t)R_2\dot{e}(t) + \alpha^T(t)\begin{bmatrix} R_2 & T \\ * & R_2 \end{bmatrix}\alpha(t) + \tau_2^2\dot{e}^T(t)R_3\dot{e}(t) + \alpha^T(t)\begin{bmatrix} R_3 & T \\ * & R_3 \end{bmatrix}\alpha(t)$$
$$+\tau_1^2\dot{e}^T(t)R_1\dot{e}(t) + \alpha^T(t)\begin{bmatrix} R_1 & T \\ * & R_1 \end{bmatrix}\alpha(t)$$
$$= \xi^T(t)\Sigma_1\xi(t) + \xi^T(t)\Sigma_2\xi(t) + \xi^T(t)\Sigma_3\xi(t) = \xi^T(t)(\Sigma_1+\Sigma_1+\Sigma_3)\xi(t)$$
$$\le \xi^T(t)\Sigma_1\xi(t)$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & \Sigma_{19} & \Sigma_{110} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26} & \Sigma_{27} & \Sigma_{28} & \Sigma_{29} & \Sigma_{210} \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} & \Sigma_{37} & \Sigma_{38} & \Sigma_{39} & \Sigma_{310} \\ * & * & * & \Sigma_{44} & \Sigma_{45} & \Sigma_{46} & \Sigma_{47} & \Sigma_{48} & \Sigma_{49} & \Sigma_{410} \\ * & * & * & * & \Sigma_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Sigma_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Sigma_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \Sigma_{1010} \end{bmatrix} < 0 \tag{24}$$

Where

$\Sigma_{11} = -A_1P_1 + B_1Y - A_1^TP_1^T + B_1^TY^T + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + M_{11}^T + M_{11} + M_{12}^T + M_{12} - T + H^TH$,

$\Sigma_{12} = A_3P_1 - KB_2Y - A_1^TP_1^T + B_1^TY^T + R_2 - T - M_{11}^T + M_{21}$

$\Sigma_{13}=A_2P_1-A_1^TP_1^T+B_1^TY^T-M_{12}^T+M_{22}$, $\Sigma_{14}=R_2$, $\Sigma_{15}=(A_3P_1+B_1Y)^T-A_1^TP_1^T+B_1^TY^T$, $\Sigma_{16}=\tau_1(A_3P_1+B_1Y)^T-A_1^TP_1^T+B_1^TY^T$ $\Sigma_{17}=\tau_2(A_3P_1+B_1Y)^T-A_1^TP_1^T+B_1^TY^T$, $\Sigma_{18}=\tau_1S_1$ $\Sigma_{19}=\tau_2S_2$, $\Sigma_{110}=P_1C_1$, $\Sigma_{22}=A_1P_1-(1-\tau)M_{11}^T+(1-\tau)M_{22}$, $\Sigma_{23}=0$, $\Sigma_{24}=R_2-T$ $\Sigma_{25}=(A_1P_1)^T$, $\Sigma_{26}=\tau_1(A_1P_1)^T$ $\Sigma_{27}=\tau_2(A_1P_1)^T$, $\Sigma_{28}=\tau_2S_3$, $\Sigma_{29}=0$, $\Sigma_{210}=0$, $\Sigma_{33}=-Q_2-(1-\tau)M_{22}^T-(1-\tau)M_{22}$, $\Sigma_{34}=0$, $\Sigma_{35}=(B_1P_1)^T$ $\Sigma_{36}=\tau_1(B_1P_1)^T$, $\Sigma_{37}=\tau_2(B_1P_1)^T$, $\Sigma_{38}=0$, $\Sigma_{39}=\tau_2R_3$, $\Sigma_{310}=0$, $\Sigma_{44}=-P_1$, $\Sigma_{45}=B_1^T$, $\Sigma_{46}=B_2^T$, $\Sigma_{47}=\tau_2B_1^T$, $\Sigma_{48}=\tau_2B_2^T$, $\Sigma_{49}=0$, $\Sigma_{410}=D_1$, $\Sigma_{55}=-\tau R_3$ $\Sigma_{66}=-\tau_1R_1$, $\Sigma_{77}=-\tau_2R_2$, $\Sigma_{88}=-\tau_1R_1^T$, $\Sigma_{99}=-\tau_2R_2^T$, $\Sigma_{1010}=-\gamma^2I$, $\alpha(t)=[\alpha_1^T(t),\alpha_2^T(t),\alpha_3^T(t),\alpha_4^T(t)]^T$

$\alpha_1(t)=e(t-\tau(t))-e(t-\tau),\alpha_2(t)=e(t-\tau(t))-e(t-\tau)-\dfrac{2}{t-\tau(t)}\int_{t-\tau}^{t-\tau(t)}e(s)ds$,

$\alpha_3(t)=e(t)-e(t-\tau(t)),\alpha_4(t)=e(t)-e(t-\tau(t))-\dfrac{2}{\tau(t)}\int_{t-\tau(t)}^{t}e(s)ds$

$\xi(t)=\left[e^T(t),e^T(t-\tau(t)),e^T(t-\tau),e^T(t-\tau_1(t)),e^T(t-\tau_1),e^T(t-\tau_2(t)),e^T(t-\tau_2),g^T(e(t)),\hat{x}(t-\tau(t)),w^T(t)\right]^T$

Combining the above results, $\dot{V}(e(t))$ can be obtained:

$$\begin{aligned}\dot{V}(e(t))&\le -e^T(t)[P_1(A_1(t)+KB_1(t))+(A_1(t)+KB_1(t))^TP_1]e(t)+2e^T(t)P_1(A_3(t)-KB_2(t))e(t-\tau_1(t)-\tau_2(t))\\&+2e^T(t)P_1A_2(t)g(e(t))+2e^T(t)P_1(C(t)-KD(t))w(t)+2g^T(e_i(t))\Lambda\dot{e}(t)+e^T(t)(Q_1+Q_2+Q_3)e(t)\\&-e^T(t-\tau_1)Q_1e(t-\tau_1)-e^T(t-\tau_2)Q_2e(t-\tau_2)-e^T(t-\tau)Q_3e(t-\tau)-(1-\mu_1)e^T(t-\tau_1(t))Q_1e(t-\tau_1(t))\\&-(1-\mu_2)e^T(t-\tau_2(t))Q_2e(t-\tau_2(t))-(1-\mu)e^T(t-\tau(t))Q_3e(t-\tau(t))+e^T(t)R_1e(t)+g^T(e(s))R_3g(e(s))\\&+2e^T(t)R_2g(e(s))-(1-\mu)e^T(t-\tau(t))R_1e(t-\tau(t))-(1-\mu)g^T(e(t-\tau(t)))R_3g(e(t-\tau(t)))\\&-2(1-\mu)e^T(t-\tau(t))R_2g(e(t-\tau(t)))+\xi^T(t)(\Sigma_1+\Sigma_2+\Sigma_3)\xi(t)\\&\le\xi^T(t)(\Sigma_1+\Sigma_2+\Sigma_3)\xi(t)\\&\le\xi^T(t)\Sigma\xi(t)<0\end{aligned}\tag{25}$$

Therefore, if LMIs is to be work, then $\dot{V}(e(t))$ <0. The neural networks (1) is asymptotically stable. This completes the Proof 1.

Since the function $f(x(t))$ satisfy (7). Then, for any $e(t)\neq 0$.

$$0\le\frac{g_i(e(t),\hat{x}(t))}{e(t)}=\frac{f(x(t))-f(\hat{x}(t))}{x(t)-\hat{x}(t)}\le l_i\tag{26}$$

Under the zero-initial condition, it is obvious that $V(e(t))|_{t=0}=0$. For dealing easily. Let

$$J_\infty=\int_0^t[\bar{z}^T(s)\bar{z}(s)-\gamma^2w^T(s)w(s)]ds,t>0\tag{27}$$

Then,

$$J_\infty\le\int_0^t[\bar{z}^T(s)\bar{z}(s)-\gamma^2w^T(s)w(s)]ds+V(e(t))-V(e(t))\big|_{t=0},t>0\tag{28}$$

Then for any $w(t)\in L^2[0,\infty]$,

$$J_\infty\le\int_0^t p[\bar{z}^T(s)\bar{z}(s)-\gamma^2w^T(s)w(s)+\dot{V}(e(s))]ds\tag{29}$$

Based on the equalities and inequalities, we can deduce that:

$$\bar{z}^T(t)\bar{z}(t) - w^T(t)w(t) + \dot{V}(e(t)) \le \xi^T(t)\left[\Sigma_1 + \tau^2\Sigma_2^T R_2\Sigma_2\right]\xi(t) \tag{30}$$

So if $\Sigma_1 + \tau^2{\Sigma_2}^T R_2\Sigma_2 < 0$, then there must exist a sufficiently small scalar σ, so $\Sigma_1 + \tau^2{\Sigma_2}^T R_2\Sigma_2 + \sigma I \le 0$. Then, it is easy to obtain

that for any $w(t) \ne 0$.

$$\begin{aligned}\bar{z}^T(t)\bar{z}(t) - w^T(t)w(t) + \dot{V}(e(t)) &\le \xi^T(t)[\Sigma_1 + \tau^2{\Sigma_2}^T R_2\Sigma_2]\xi(t) \\ &\le -\sigma\xi^T(t)\xi(t) \le -\sigma w^T(t)w(t) < 0\end{aligned} \tag{31}$$

Proof 2. The system is robustly, asymptotically stable if the following inequalities is satisfied:

$$\Omega + \varphi_1 F(t)\varphi_2^T + \varphi_2 F(t)\varphi_1^T + \varphi_3 F(t)\varphi_4^T + \varphi_4 F(t)\varphi_3^T + \varphi_5 F(t)\varphi_6^T + \varphi_6 F(t)\varphi_5^T < 0 \tag{32}$$

Where $\varphi_1 = \begin{bmatrix} H^T P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $\varphi_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H^T R_3 \end{bmatrix}^T$, $\varphi_3 = \begin{bmatrix} M_{11} & 0 & M_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $\varphi_4 = \begin{bmatrix} M_{21} & 0 & M_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $\varphi_5 = \begin{bmatrix} S_1 & 0 & S_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, $\varphi_6 = \begin{bmatrix} S_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$

If (13) is satisfied, then the following inequalities:

$$\Omega + \varepsilon_1^{-1}\varphi_1\varphi_1^T + \varepsilon_1\varphi_2\varphi_2^T + \varepsilon_2^{-1}\varphi_3\varphi_3^T + \varepsilon_2\varphi_4\varphi_4^T + \varepsilon_3^{-1}\varphi_5\varphi_5^T + \varepsilon_3\varphi_6\varphi_6^T \equiv \Omega + \psi$$

$$= \Omega + \begin{bmatrix} \begin{matrix}\varepsilon_1^{-1}P_1^T HH^T P_1 + \varepsilon_2^{-1}M_{11}M_{11}^T \\ + \varepsilon_2 M_{21}M_{21}^T + \varepsilon_3^{-1}S_1S_1^T + \varepsilon_3 S_3S_3^T\end{matrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & \begin{matrix}\varepsilon_2^{-1}M_{12}M_{12}^T + \varepsilon_2 M_{22}M_{22}^T \\ + \varepsilon_3^{-1}S_2S_2^T\end{matrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & \varepsilon_1 R_4 H^T H R_4^T \end{bmatrix} \tag{33}$$

where $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0$.

Then, the inequalities given in (33) is equivalent to the LMIs (25). Thus, if the LMIs given in (25) is satisfied, then the system (10) is robust asymptotically stable. This completes the Proof 2.

Corollary 1 Considering the neural networks with two additive time-varying delays system (1), for given scalars $\mu < 1$ and K, let γ be a prescribed constant, the guaranteed $H\infty$ performance state estimation problem is solvable if there exist real matrices $Y > 0$, $M_{je} > 0\,(j = 1,2, e = 1,2)$; and diagonal matrices $P_2 = diag\{\eta_i\}$ with appropriate dimensions, then the following LMIs are satisfied:

$$\Phi = \begin{bmatrix} \Phi_{11} & \begin{matrix} A_3 Y \\ -B_2 Y \end{matrix} & \Phi_{13} & P_1 & P_1 B_1 - Y B_2 & H^T \\ * & -(1-\mu)M_{11} & 0 & 0 & 0 & 0 \\ * & * & -(1-\mu)M_{12} & \Phi_{34} & 0 & 0 \\ * & * & * & -(1-\mu)M_{21} & 0 & 0 \\ * & * & * & * & -(1-\mu)M_{22} & 0 \\ * & * & * & * & * & -\gamma I \end{bmatrix} \quad (34)$$

$\Phi_{11} = -P_1 A_1 - A_1^T P_1^T - B_1 Y - Y^T B_1^T + M_{11} + M_{12} + M_{21} + M_{22}$

$\Phi_{13} = -(1-\mu)(M_{11} + M_{12}) - 2P_2$, $\Phi_{34} = -(1-\mu)(M_{21} + M_{22}) - 2P_2$

Moreover, the gain matrix K of the state estimator of (8) can be designed as $K = P_1^{-1} Y$.

Remark 1. Theorem 1 provides a novel robust $H\infty$ state estimation of uncertain neural networks stability criterion for system (1) with two additive time-varying delays components, it has been verified by a form of less complex LMIs.

Remark 2. Based on the Lyapunov stability criterion, a novel LMIs is constructed to prove that the derivative of the LKFs is smaller than zero, and that the system (1) is asymptotically stable.

Remark 3. The state estimation uncertain neural networks methods are listed in table 1.

Table 1. state estimation of uncertain neural network methods.

numbers	1	2	3	4	5	6	7
methods	$H\infty$ State estimation [22]	Robust finite-time state estimation [21]	State estimation [26]	Extended dissipative state estimation [27]	Delay dependent state estimation [28]	Non-fragile state estimation [29]	Recursive state estimation [30]

Compared with other theories, the $H\infty$ theory can give delay-dependent criteria, so that the error system has globally asymptotic stability $H\infty$ performance.

Remark 4. The structure of paper is organized as follows: In Section 2, problem model are given. In Section 3, a new theorem and three corollaries are established. In Section 4, two simulation results are provided to demonstrate the effectiveness of the developed approach. Finally, Section 5 summarizes this work.

4. Numerical Examples

Two numerical simulation examples are to be presented to show the feasibility of the developed approach.

Example1. Considering the system (10) with following parameters:

$A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$, $A_3 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $x_{delay} = \begin{bmatrix} 0 & 0 & 0.2 \end{bmatrix}$, $x = \begin{bmatrix} 0.2 \\ 0.3 \\ 1 \end{bmatrix}$, $\tau_1 = 0.15, \tau_2 = 0.24$, $t = 0, dt = 0.001$.

In addition, the activation function is chosen as $f(x)=\tanh(x)$, the time-varying delay by $\tau(t)=0$. It is easy to get $K=\begin{bmatrix}-4.0107 & -1.7896\\ -1.7896 & -4.6767\end{bmatrix}$, $\tau=0.39$ .the noise disturbance is assumed be $w(t)=\dfrac{1}{1.2t+0.8}$.

$P_1=\begin{bmatrix}0.6749 & -0.0197\\ -0.0197 & 0.5245\end{bmatrix}$, $P_2=\begin{bmatrix}0.1972 & 0\\ 0 & 0.1972\end{bmatrix}$, $Y=\begin{bmatrix}-4.0107 & -1.7896\\ -1.7896 & -4.6767\end{bmatrix}$, $S_1=\begin{bmatrix}-2.8839 & 0.0969\\ 0.0969 & -2.4532\end{bmatrix}$, $S_2=\begin{bmatrix}-0.0163 & 0.0291\\ 0.0291 & 0.0200\end{bmatrix}$,

$S_3=\begin{bmatrix}0.0250 & -0.0874\\ -0.0874 & -0.0408\end{bmatrix}$, $Q_1=\begin{bmatrix}581.417 & 58.2298\\ 58.2298 & -436.2641\end{bmatrix}$,

$Q_2=\begin{bmatrix}0.8732 & 0.1111\\ 0.1111 & 1.0659\end{bmatrix}$, $Q_3=Q_4=Q_5=Q_6=\begin{bmatrix}-436.4433 & -57.6238\\ -57.6238 & 64.8827\end{bmatrix}$, $M_{11}=\begin{bmatrix}282.9611 & 44.2305\\ 44.2305 & 37.1869\end{bmatrix}$, $M_{12}=\begin{bmatrix}300.9950 & 43.6859\\ 43.6859 & 53.3194\end{bmatrix}$,

$M_{21}=\begin{bmatrix}280.9950 & 42.1068\\ 42.1068 & 33.6494\end{bmatrix}$, $M_{22}=\begin{bmatrix}300.6336 & 43.8862\\ 43.8862 & 53.7267\end{bmatrix}$

With the option $H\infty$ performance index $\gamma=1.3811$. Fig 1 shows the responses of the estimation error curves which generated by random initial value, it confirms the feasibility of the developed LMIs method through the designed state estimator of uncertain neural networks with two additive time-varying delays.

By applying the MATLAB LMI toolbox, it is found that LMIs (13) is feasible. As Fig.1 illustrates, the initial matrix of the example1 is a two-dimensional matrix, there are four estimation error curves. The state estimation tends to 0 rapidly. Therefore, it is proved that uncertain neural networks with two additive time-varying delays is globally asymptotically stable through proposed LMIs. The calculated minimum $H\infty$ performance index γ with different $\mu<1$ values are listed in Table 2. One can know clearly that the results obtained by Corollary 1 can provide smaller $H\infty$ performance index γ than recently existing method in [1].

Table 2. Minimum $H\infty$ performance index γ with different μ.

μ	0.4	0.5	0.7
Reference [1]	0.4632	0.5301	1.3819
Corollary 1	0.0035	0.1544	0.4402

Example2. Considering a delayed neural network of Corollary 1 with following parameters:

$A_1=\begin{bmatrix}-1 & 1 & 0\\ 2 & 1 & -1\\ 1 & -1 & 2\end{bmatrix}$, $B_1=\begin{bmatrix}-1 & 1 & 0\\ -1 & 1 & -1\\ -1 & -1 & 1\end{bmatrix}$, $B_2=\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix}$, $A_3=\begin{bmatrix}-1 & 0 & 0\\ 1 & 1 & 0\\ 1 & -1 & -1\end{bmatrix}$, $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, $\gamma=0.5, \mu=0.5$

In addition, the activation function is chosen as $f(x)=\tanh(x)$, the time-varying delay by $\tau(t)=0$. It is easy to get

$K=10^{-9}*\begin{bmatrix}0.0120 & 0.0394 & 0.0110\\ 0.0394 & 0.2970 & -0.1812\\ 0.0110 & -0.1812 & 0.2188\end{bmatrix}$, $P_1=10^{-9}*\begin{bmatrix}-0.0245 & 0.0744 & 0.0188\\ 0.0744 & 0.2718 & -0.1799\\ 0.0188 & -0.1799 & 0.2296\end{bmatrix}$, $P_2=10^{-10}*\begin{bmatrix}-0.1238 & 0 & 0\\ 0 & -0.1238 & 0\\ 0 & 0 & -0.1238\end{bmatrix}$,

$Y=10^{-9}*\begin{bmatrix}0.0120 & 0.0394 & 0.0110\\ 0.0394 & 0.2970 & -0.1812\\ 0.0110 & -0.1812 & 0.2188\end{bmatrix}$, $H=10^{-11}*\begin{bmatrix}0.6127 & 0 & 0\\ 0 & 0.6127 & 0\\ 0 & 0 & 0.6127\end{bmatrix}$, $M_{11}=10^{-9}*\begin{bmatrix}0.0244 & -0.0197 & -0.0132\\ -0.0197 & 0.0939 & -0.1353\\ -0.0132 & -0.1353 & 0.1524\end{bmatrix}$,

$M_{12}=10^{-9}*\begin{bmatrix}0.0400 & 0.0266 & -0.0520\\ 0.0266 & 0.1108 & -0.1033\\ -0.0520 & -0.1033 & 0.1156\end{bmatrix}$, $M_{21}=10^{-9}*\begin{bmatrix}0.0727 & 0.0332 & -0.0076\\ 0.0332 & 0.1481 & -0.0368\\ -0.0076 & -0.0368 & 0.1161\end{bmatrix}$, $M_{22}=10^{-10}*\begin{bmatrix}0.1040 & -0.1395 & -0.1606\\ -0.1395 & -0.3238 & 0.0840\\ -0.1606 & 0.0840 & -0.0893\end{bmatrix}$.

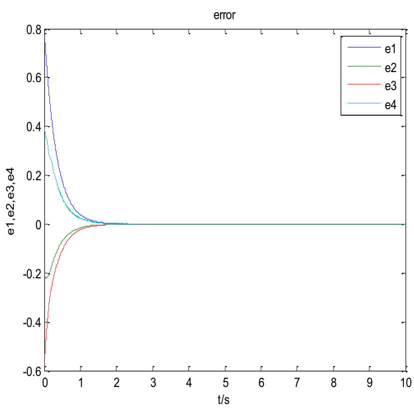

Figure 1. Estimation errors e_1, e_2, e_3 and e_4

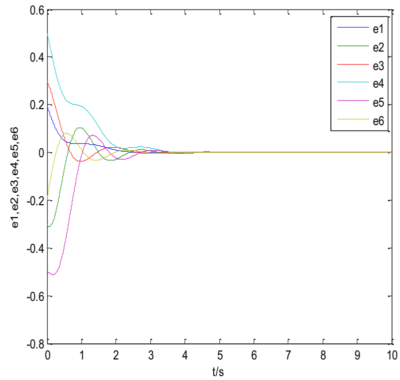

Figure 2. Estimation errors e_1, e_2, e_3, e_4, e_5 and e_6

The global asymptotically stable simulation results of the system (10) with the above parameters is illustrated in Fig.2.
the initial matrix of the example 2 is a three-dimensional matrix, there are six estimation error curves.

5. Conclusions

In this paper, the problem of $H\infty$ state estimation of uncertain neural networks with two additive time-varying delays has been studied. Based on LKFs method, a novel LMIs method has been established to ensure the global asymptotic stability of uncertain neural networks with two additive time-varying delays. It has fine convergence speed through constructed LMIs. Two numerical simulation examples have been performed to demonstrate the feasibility of the developed approach.
We would like to point out that this work did not include the $H\infty$ state estimation of uncertain neural networks with leaking time delay.

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