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Investigation of a Method for EEG Signal De-Noising Based on the DIVA Model

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Abstract. The DIVA (Directions Into Velocities of Articulators) model is an adaptive neural network model that is used to control the movement of the analog vocal tract to generate words, syllables, or phonemes. The input signal to the DIVA model is the EEG (electroencephalogram) signal acquired from the human brain. However, due to the influence of power frequency interference and other forms of noise, the input signal can be non-stationary and can also contain a variety of multi-form waveforms in its instantaneous structure. Input of such a signal into the DIVA model affects normal speech processing. Therefore, based on the concept of sparse decomposition, this paper applies and improves an adaptive sparse decomposition model for feature extraction of the general EEG signal structure and then uses the Matching Pursuit algorithm to compute the optimal atom. The original EEG signal can then be represented by atoms in a complete atomic library. This model removes noise from the EEG signal resulting in a better signal than the wavelet transform method. Finally, applies the EEG signal denoised by this model to DIAV model. Simulation results show that the method improves phonetic pronunciation greatly.

Keywords. DIVA model, EEG signal, noise, sparse decomposition

1. Introduction

Research works on various fields (phonetics, control science, robotics, neural physiology) is required in order to accurately simulate and describe functions of brain regions responsible for speech acquisition and production based on neurophysiology and neuroanatomy. A new tool, called the Neuralynx System[1], developed by a team led by Frank Guenther of Boston University, is the most representative and successful among them. The primary feature of this tool is that users only need to think about what they want to express and the speech synthesis system converts their thoughts into speech. The principle is shown in Figure 1.

In Figure 1, black circles and curved arrows represent neurons and axonal projections in the neural circuit for speech motor output, respectively. Signals collected from an electrode implanted in the subject's speech motor cortex are amplified and transmitted wirelessly across the scalp as FM radio signals. The signals are routed to an electrophysiology recording system for further amplification, analog-to-digital

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conversion, and spike sorting. The sorted spikes are sent to a neural decoder which translates them into commands for a speech synthesizer. Audio signals from the synthesizer are fed back to the subject in real time (PrCG represents the precentral gyrus in the brain).

Figure 1. Schematic diagram of Neuralynx System

The Neuralynx System consists of two parts: brain computer interface (BCI) and speech synthesis system DIVA model (Directions Into Velocities of Articulators). BCI is used to achieve communication and control between a human brain and a computer or other electronic device. Input signal is generated from a permanently implanted wireless neural electrode in the cerebral cortex of an aphasia patient that detects generated speech and obtains neural signals from related areas [1, 2]. These signals drive the speech synthesis system to "operate" continuously and provide the patient with real-time speech output. The DIVA model is a biological neural network for generating and obtaining speech [3].

The EEG input signal acquisition process for the DIVA model includes conditioning, sampling, quantization, coding, and transmission. During this process, the EEG signal is non-stationary and can be corrupted by various forms of noise, particularly frequency interference. Input of such an EEG signal into the DIVA model would affect normal speech processing. Therefore, it is necessary to eliminate noise in the original EEG data.

Current de-noising methods include notch filtering, adaptive filtering, and wavelet transform, to name a few. Notch filter leads to EEG waveform distortion [4]. The adaptive filter can automatically track the frequency change for power frequency interference and minimize the loss of useful information, but the frequency tracking range is narrow. The wavelet transform is the most widely used; however, it has drawbacks. For example, the calculation is complex and the choice of wavelet basis and wavelet threshold requires prior knowledge [5-6].

With this in mind, Mallat and Zhang proposed sparse decomposition [7] based on an over-complete dictionary. Based on signal characteristics, sparse signal decomposition can adaptively select the appropriate basis functions to complete signal decomposition. In this process, the over-complete dictionary plays a key role. This paper improves a construction method for an over-complete dictionary by analyzing the structural characteristics of EEG signals and using the matching pursuit algorithm [7, 8] (MP) for sparse decomposition followed by reconstitution. Following this process, the proposed algorithm enhances EEG signal sparsity, performs de-noising, and improves

the speech processing ability of the DIVA model.

2. Diva Model

The DIVA model is an adaptive neural network that describes the processes of speech acquisition and production and generates speech by controlling the simulated voice channel [9,10]. The model is based on behavioral data collected from physical experiments in speech generation and sensory psychology, neuroimaging fMRI data (functional magnetic resonance imaging) and PET (positron emission computed tomography) experiments, and neurophysiology data from motion control experiments in animals. The principle is shown in Figure 2[3].

As shown in Figure 2, the DIVA model consists of a feedforward control subsystem, a feedback control subsystem, and a simulated Maeda vocal tract. By recording the input speech formant frequency during training, the model generates a phonating rate and a time variable sequence that represents positional variations in vocal organs. The model uses this sequence to obtain the required phonations. The feedforward control system is responsible for speech production and the feedback control system is responsible for speech learning. In the feedforward control system, the generation of a phoneme or syllable begins with the activation of a corresponding cell speech map set. Each cell corresponds to a single phoneme or syllable.

Figure 2. Working wireframes of DIVA model

The DIVA model and fMRI are very closely linked [11]. Various assumptions made by the DIVA model can be tested and demonstrated by applying fMRI experiments. Data obtained from fMRI can also be analyzed and interpreted by the DIVA model. Thus, the DIVA model is a basic framework which can interpret speech neural processes.

3. Sparse Signal Decomposition

The primary aims of signal sparse decomposition are: (1) decompose the signal in the over-complete dictionary, (2) select the base function of the signal adaptively based on signal structural characteristics, and (3) compute correlation coefficients to have only a few non-zero values. Signal sparse decomposition has been successfully applied in many aspects of signal processing, such as signal detection, signal recognition, and image de-noising. A variety of sparse decomposition algorithms have been developed: MP algorithm (Matching Pursuit) [7,8], BP algorithm (Basis Pursuit)[12], BOB algorithm (Basis Orthogonal Best) [13], and OMP algorithm[14] (Orthogonal Matching Pursuit). BP, and especially MP are the most commonly used.

Figure 3. Schematic diagram of neuroanatomy mapping relation to the DIVA model

3.1. MP Algorithm

The Matching Pursuit algorithm (MP) is a signal analysis method that was proposed by Mallet and Zhang in 1993 and belongs to a category of greedy algorithms.

The basic idea is to select the largest component of the correlation coefficients through decomposing the signal in the library (over-complete dictionary). The algorithm gets the sparse representation of the signal through multiple iterative decompositions [15,16]. Implementation of the algorithm can be described as follows:

First, select an over-complete dictionary where $D = \{gr, r = 1, 2, ..., M\}$ represents such a collection. Elements of the collection are termed atoms. Each atom can adequately represent the characteristics of the signal. Maintain the over-complete feature amongst the atoms. The so-called overcompleteness is the inner product $\langle g, g \rangle \neq 0$ between two different atoms gi and gi, where $i\neq j$.

The MP algorithm makes the inner product the largest between signal f and the atoms of the over-complete dictionary. This characteristic is regarded as the optimization principle of greedy algorithms [6].

First, select the best atom gi from dictionary D which satisfies the following condition:

$$
\left| \left\langle f, g_i \right\rangle \right| = \sup_{j \in M} \left| \left\langle f, g_j \right\rangle \right| \tag{1}
$$

where $|\langle f, g \rangle|$ is the inner product between signal f and atom gi.

After selecting gi, the signal f is decomposed as follows:

$$
f = \langle f, g_i \rangle g_i + R^1 f \tag{2}
$$

where $\leq f$, g_i is the projection of signal f onto the atom g_i , and $R¹f$ is the residual value after projecting signal f onto gi (called residual error).

Now, remove the atom gi from the initial over-complete dictionary because gi has been used as part of the signal f and will not be used to find the matching atom afterwards; this reduces the amount of computation. Decompose the residual error from

signal f and determine the best atom in the revised over-complete dictionary. The formula can be expressed as follows:

$$
R^1 f = \langle R^1 f, g_k \rangle g_k + R^2 f \tag{3}
$$

Of course, g_k still needs to be determined as an optimum atom:

$$
\left| \left\langle R^1 f, g_k \right\rangle \right| = \sup_{j \in M} \left| \left\langle R^1 f, g_j \right\rangle \right| \tag{4}
$$

After this iterative process, the signal f can be decomposed into:

$$
f = \sum_{L=0}^{L-1} \left\langle R^l f, g_l \right\rangle g_l + R^L f \tag{5}
$$

Mallet proved that the value of $||R^Lf||$ exponentially converges with increasing L. Thus, the signal f can be approximated by the following decomposition:

$$
f \approx \sum_{l=1}^{L-1} \langle R^l f, g_l \rangle g_l \tag{6}
$$

The steps above are shown below in Figure 4:

Figure 4. Flow diagram of MP algorithm

3.2. Constructing the Over-complete Dictionary

The Matching Pursuit algorithm decomposes the signal based on a complete library. Constructing the over-complete dictionary is the critical step[17]. This paper constructs a specific over-complete dictionary that would be applicable to EEG signals.

As noted earlier, EEG signals are often corrupted by power frequency interference and other forms of noise during acquisition. This makes the EEG signal non-stationary and yields a variety of multi-form waveforms in its instantaneous structure. Thus, atomics of the single structure type are unable to match the transient EEG waveform effectively. In order to match multi-form waveforms in its instantaneous structure of EEG, the atomic dictionary should contain a variety of structural atoms. In order to extract the feature of EEG signal, reference [18] proposed an SSDM (Structure

Adaptive Sparse Decomposition Model) which is suitable for EEG signal in patients with psychomotor epilepsy. We studies and improves the SSDM model to extract the feature of EEG signal in normal people. The improved SSDM model aims to detect the spike wave automatically. From the view of matching multi-form waveforms in its instantaneous structure of EEG signal, improved SSDM model applies multi-structural atoms to adaptively depose and analyze the general EEG signal which is more suitable for detecting the instantaneous structure of EEG signal. General EEG signal consists of positive phase and negative phase. Small scale Gaussian function can match the single - phase spike wave well and large scale Gaussian function can represent the low frequency component of signal well. Moreover the Gaussian wavelet can match the double-phase spike wave well. We use Gaussian function and its first order derivative as the generating function to design a new multi-component dictionary which differs from the original SSDM model.

$$
\varphi_r^1(t) = K_1(r) \exp\left\{-\frac{(t-u)^2}{2s^2}\right\}
$$

$$
\varphi_r^2(t) = K_2(r) \left(\frac{t-u}{s}\right) \exp\left\{-\frac{(t-u)^2}{2s^2}\right\}
$$
(7)

where $K1(r)$, $K2(r)$ are normalization factors that allow atoms to show the unitized norm, $\varphi_r^1(r)$, $\varphi_r^2(r)$ are Gaussian functions and their first derivatives, and parameter set $r = \{u, s\}$ indicates location and scale characteristics of atoms, respectively. By using transforming methods such as panning and stretching, free variables u and s can be modulated to generate a series of atoms that form a variety of transient and redundant databases which can match the multi-constituent structure.

In reality, free parameters must be sampled to form a discrete atom dictionary in order to sparsely decompose into a discrete digital signal. Discretized atoms may be expressed as:

$$
\varphi_r^2[n] = K_1(r) \exp\left\{-\frac{(n-p)^2}{2(a^i)^2}\right\}
$$

$$
\varphi_r^2[n] = K_2(r)\left(\frac{n-p}{a^i}\right) \exp\left\{-\frac{(n-p)^2}{2(a^i)^2}\right\}
$$
(8)

where both p and i are integers, $r = \{p, ai\}$, $p \in [0, N-1], i \in [0, \log_l N]$ is the discrete set of parameters, and N is the dimension of discrete signals to be decomposed. Generally, a $= 2$ is desired.

The atomic dictionary generated by this improved SSDM is complete and posses the invariance under translation in time and approximate invariance in scale. Moreover the numbers of atomic in improved SSDM are fewer than those in Gabor, which makes the improved SSDM have lower searching complexity and higher sparse decomposing efficiency. This improved SSDM model regard the Gaussian function and Gaussian wavelet as atomic dictionary generating functions which makes the atom structure match the separated instantaneous structure of general EEG signal more closely.

In frequency analysis of the EEG signal, time-frequency structural parameters obtained after sparse decomposition should establish direct contact with the artificial vision analysis criteria. Comparing these time-frequency structural parameters with prior parameters can directly determine whether it is a characteristic EEG signal waveform. The discrete atomic dictionary designed by this method has explicit morphological structure parameters, such as location, degree, and amplitude.

4. Simulation Experiment

4.1. De-noising Principle

The signal can be separated into the original signal and the noise signal based on whether the correlation coefficient is zero after sparse decomposition. Suppose the original EEG signal with noise is:

$$
f = E + N \tag{9}
$$

where E is the original signal without noise and N is an independently distributed random noise signal. When applying the MP algorithm in the atomic dictionary, the dictionary is constructed based on the structural characteristics of the EEG signal. Therefore, atom structure must be related to the EEG signal, regardless of noise. The formula can be expressed as:

$$
f = \sum_{L=0}^{L-1} \langle R^l f, g_l \rangle g_l + R^L f \tag{10}
$$

The first part of the equation is the original EEG signal and the second part is the residual after extracting the EEG signal, i.e., the noise signal. The equation can be compared with the formula:

$$
E = \sum_{L=0}^{L-1} \langle R^l f, g_l \rangle g_l
$$

$$
N = R^L f
$$
 (11)

4.2. Experimental Process and Results Analysis

4.2.1 Experimental Design and Signal Collection

The following experimental data comes from the State Key Laboratory for Cognitive Neuroscience and Learning at Beijing Normal University, which is our collaboration unit.

The subject is a healthy man with experience in EEG acquisition experiments. The experiment uses an electrical scanner and a scanning cap with 128 electrodes to record EEG signals (Figure 5). The sampling frequency of the signal is 1000 Hz. During the process of collecting the EEG signal, the subject's consciousness was clear and he sat on an ordinary chair. The expression of the word "happy" in English was performed 100 times. This experiment was completed in one day.

4.2.2 Experimental Tools

Experimental evaluation uses the EEGlab Toolbox in MATLAB R2010b to read collected EEG signals. The EEGlab Toolbox is a tool for processing EEG data and reading the collected signal waveform.

4.2.3 Pretreating Data

While using the non-invasive acquisition method as described, EOG and muscular movement will degrade the signal. Prior to analyzing the EEG signal, we made use of the ICA (Independent Component Analysis) function in EEGlab to extract the main constituent of effective independence which can remove EOG and muscular movement from the original signal.

Figure 5. Electrode locations on scanning cap

4.2.4 Construction of SSDM Atom Dictionary and Signal Reconstruction

In sparse decomposition theory, an atomic library has good structure when the following is satisfied: (1) the atomic dictionary contains the most atomic numbers and types possible in order to achieve sparse decomposition and a good sparse decomposition effect and (2) the atomic dictionary doesn't use similar atoms for both storage and computation. When these two criteria are maintained, a good balance is achieved and the structure of the atom dictionary is optimal.

The atom dictionary described above can be used to obtain general atoms for use in sparse decomposition. Figure 6 is a shape schematic for two atoms in which atomic length is 1024. The figure shows concentrated energy in the central region and zero elsewhere. These are representative atoms. Results are consistent with previous work [7].

The following verifies the signal reconstruction effect using the SSDM atom dictionary and contrasts it with the Gabor dictionary. First, we intercept the normal EEG signal of the Chinese vowel /a/ after filtering through an EEG database provided by the Brain Research Center at Beijing Normal University.

Figure 7(a) is a normal brain waveform of the Chinese vowel /a/, which is smooth under normal conditions. Figure 7(b) is the waveform graph of the Gabor atom reconstructed vowel /a/ and Figure 7(c) shows the result of the SSDM atom reconstructed vowel /a/.The decomposition and reconstruction of the EEG signal using the SSDM atom dictionary compares well with the Gabor atom dictionary.

Since the morphological structure of the atom in SSDM matches each transient structure of the EEG signal more closely, atomic numbers are fewer in SSDM compared with those from the Gabor dictionary. Sparse decomposition efficiency is also improved in SSDM. Table 1 shows the average running time for the signal to reach the optimal result in both methods. Table 1 shows that calculation time is reduced and speed is greatly improved in SSDM.

Figure 7(c). SSDM dictionary reconstructed result Figure 8. Attenuation graphs of MMSE with an

increasing numbers of atoms in the two atom libraries

4.2.5 Comparison of the Matching Pursuit Sparse Approximation Between the SSDM and Gabor

In order to compare the performance of matching pursuit sparse approximation between the SSDM and Gabor, NMSE (normalized mean square error) is used. The formula is:

$$
NMSE = \frac{\left\| s - s^{\cdot} \right\|_{2}^{2}}{\left\| s \right\|_{2}^{2}}
$$
\n(12)

Figure 8 is the attenuation graphs of MMSE with an increasing numbers of atoms in the two atom libraries. It shows that the improved SSDM decays more quickly. Since the improved SSDM consists of multi-structure atoms which can match multi-form waveforms in its instantaneous structure of EEG signal, it owns stronger ability to sparse approximation.

4.2.6 Comparison of the Signal De-noising Effect

Using EEGlab, we can obtain whole pronunciation waveforms during the 2-cycle (Figure 9, provided by the Institute of Cognitive Neuroscience and Learning, Beijing Normal University). We intercept one pronunciation. Since the sampling frequency is 1000 Hz and a pronounced duration is 2 sec, the signal has 2000 sampled points, as shown in Figure 10(a). We now add 50 Hz power frequency interference in which SNR (Signal to Noise Ratio) is 10 dB, 5 dB, -5 dB, and -10 dB, respectively, forming an original signal with noise. Figures $10(b)$, $10(c)$ and $10(d)$ are waveforms processed through the improved SSDM atom library, Gabor atom library and traditional wavelet transform method. It shows that the effect of de-noising based on the two atom libraries is better than wavelet transform method. Moreover the de-noising result of the SSAM is better compared to Gabor atom library. The wavelet transform removes effective constituents as noise.

The waveform obtained through the SSDM atom library maintains the original waveform component better. In order to assess the three de-noising methods, we evaluate RMSE (Root Mean Square Error) and the effect of SNR. The formula is:

$$
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (X_1(n) - X_2(n))^2}
$$

\n
$$
SNR = 10 \log_{10} \left[\sum_{n=0}^{N-1} \frac{X_1^2(n)}{(X_1(n) - X_2(n))^2} \right]
$$
\n(13)

 Signal to Noise Ratio and Root Mean Square Error of the SSDM method. SNR2 and where $X1(n)$ is the input signal, $X2(n)$ is the output signal, and N is the dimension of the signal. Table 2 shows the results of the comparison. SNR1 and RMSE1 are the RMSE2 are for the Gabor method while SNR3 and RMSE3 are for the wavelet transform method.

Table 2. SNR and RMSE for the SSDM,Gabor and wavelet transform methods.

	SNR1	SNR ₂	SNR3	RMSE1	RMSE2	RMSE3
10dB	18 9209	15 9281	14.1826	0.0015	0.0103	0.0181
5dB	20 74 10	18 2377	15 1293	0.0014	0.0099	0.0192
-5dB	23.3201	22.3230	14 0291	0.0016	0.0143	0.0314
-10dB	18 4029	15.2915	16.1082	0.0025	0.0348	0.0532

Table 2 shows that to different intensity noise, the SNR of SSDM and Gabor improve compared with wavelet method. Furthermore, the RMSE of SSDM reduces which means that the EEG signal recovered by SSDM is highly similar to the original EEG signal. Overall, the order of de-noising quality is: SSDM>Gabor>wavelet.

4.3. Speech Processing Ability of the Improved DIVA Model

4.3.1 DIVA Model Interface

When applying the DIVA model to evaluate pronunciation function, the system presents a user interface to allow users to control the pronunciation mechanism as shown in Figure 11. The interface can be divided into three parts: control module, acoustic characterization space module, and vocal tract control module.

Figure 11. User interface of DIVA model

4.3.2 Flow Chart of DIVA Speech Sound Map Module

Figure 12 shows a flowchart of the DIVA speech sound map module, which is part of the control module section in the DIVA model interface. During a simulation experiment, relevant information must be entered through this module and the input port parameters can be modified.

Figure 12. Speech sound map module of DIVA model

4.3.3 Comparison of Result Precision

We input speech learning samples of varying difficulty before de-noising and then enter the de-noised data into the DIVA model. Figure 13 shows comparative results of phonetic pronunciation accuracy in the model. Overall, speech pronunciation accuracy of the DIVA model increases when the de-noised signal is used compared to processing the signal with noise. For example, with normal difficulty speech learning samples containing noise, pronunciation accuracy is about 80% on average, while phonetic pronunciation accuracy can reach 90% after the signal is de-noised. Noise in the EEG signal affects the speech processing ability of the DIVA model.

Besides, the experiment results show that the EEG signal de-noised by Gabor atom dictionary and wavelet can also improve speech pronunciation accuracy of the DIVA model. Since the noise reduction effect by these two methods is less than SSDM, the improved extent in speech pronunciation of the DIVA model is limited compared with the SSDM. Furthermore, atomic numbers are much more in Gabor than in SSDM dictionary which makes the de-nosing efficiency based on Gabor dictionary is lower than SSDM dictionary.

Figure13. Comparative result of speech pronunciation accuracy percent between noisy and de-noised signal

5 Conclusion

This paper studies and improves an adaptive sparse decomposition model (SSDM) which is more suitable for the general EEG signal. Moreover, it applies the EEG signal de-noised by the improved SSDM to DIAV model. Simulation results show that the proposed method removes significant noise and is able to retain active components in the EEG signal. The de-noising effect is better. Meanwhile, the proposed algorithm improves speech pronunciation accuracy of the model by using a de-noised EEG signal for learning samples. This study provides a foundation to improve the speech processing capability of the DIVA model which can better describe and simulate related functions of brain regions involved in speech production and comprehension based on neuroanatomy and neuropsychology.

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